

Profits, ‘Superstar’ Firms and External Imbalances

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Abstract

In this paper, I study the net foreign asset positions of economies with differing aggregate profit shares. I show that if firms compete oligopolistically, then economies which host a large number of very large – ‘superstar’ – firms enjoy higher aggregate profit shares. Embedding this setup in a two-country model with heterogeneous agents and non-homothetic saving behavior, I show that economies with more profitable firms feature lower autarkic interest rate and hold positive net foreign assets under financial liberalization. Calibrating the model to Germany and a Rest-of-Europe aggregate, I show that the profit share gap can explain a quarter of European imbalances in 2019.

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1 Introduction

Since the 1990s, Europe has been characterized by the development of large external imbalances, with economies emerging as either persistent lenders or borrowers. Most explanations of this pattern have focused on the ‘pull factor’ in the net borrower economies, suggesting low initial capital stocks or housing bubbles as the drivers behind the imbalances. In this paper, I document a novel stylized fact: European lenders enjoyed systematically higher aggregate profit shares and featured a higher prevalence of very large – ‘superstar’ – firms. In line with this new evidence, I complement the existing explanations with a ‘push factor’ theory of European imbalances, where imperfect competition in the goods markets and associated heterogeneous profits generate external imbalances.

Recent literature has tied the emergence of external imbalances to economies’ heterogeneous capacity to, on one hand, generate stores of value (asset supply channel), and on the other, use these for saving (asset demand channel).¹ In this paper, I argue that economic profits have implications for both. On one hand, firms that earn rents ‘restrict’ their production compared to the competitive benchmark. This implies a reduced demand for inputs – including capital – making less capital available as a store of value and thus contracting the asset supply. On the other hand, profits constitute a source of income. If propensity to save out of profits is high, then higher profits also imply a higher demand for assets. As lower supply and higher demand for assets suppress the autarkic interest rate, economies that enjoy high profit shares emerge as international lenders.

This paper makes three contributions. First, I develop a minimal two-country model where the link between the aggregate profit shares and external imbalances emerges as an equilibrium outcome. Second, I test the predictions of the model in the European context. I document novel stylized facts characterizing European imbalances: positive net foreign asset positions were associated both with the prevalence of ‘superstar’ firms, and with higher aggregate profit shares. Third, I study the economic relevance of the mecha-

¹See [Gourinchas and Rey \(2014\)](#) for an overview.

nism in a quantitative setting. I extend the model and calibrate it to match sectoral- and firm-level moments of Germany and a Rest-of-Europe aggregate. I find that the model-generated German imbalances explain a quarter of those in the data.

I begin by outlining my argument in a stylized setting. I model economic profit as arising due to oligopolistic competition following [Atkeson and Burstein \(2008\)](#). Two economies trade a non-overlapping set of varieties and compete à la Cournot. In this setup, firm-level profit shares increase in productivity. The relationship of the *aggregate* profit shares and firm-level productivities, on the other hand, is non-linear. I show that if the most productive domestic firm becomes even more productive home's aggregate profit share increases. If the least productive firm becomes more productive the aggregate profit share declines. In other words, having more extreme draws for the most productive – 'superstar' – firms results in a higher aggregate profit share.

Next, I turn to asset markets. In the stylized model, the only asset is capital. Firms hire capital until its marginal revenue product equals the rental cost. Under imperfect competition, firms internalize the fact that expanding supply reduces their prices and hire less capital, compared to the competitive benchmark. As a result, less capital is available as a store of value. Asset demand comes from households, who are divided into workers and capitalists, the latter being the recipients of the profits in the economy. Under the assumption that demand for assets is non-homothetic, with richer capitalist households seeking to hold disproportionately more assets, I derive two results. First, autarkic interest rate is lower in the country that generates a higher aggregate profit share. Second, under perfect capital mobility, such economy holds positive net foreign assets in steady state.

Having characterized the link between profits and external imbalances theoretically, I revisit European imbalances. I show that in Europe, economies with thicker tails of firm size distribution enjoyed higher aggregate profit shares – consistent with oligopolistically competitive behavior. Moreover, both the thickness of the tails and aggregate profit shares were associated with higher net foreign asset positions, consistent with the predictions of

the model. I show that these results are robust to controlling for other possible drivers of external imbalances and to including non-European economies in the sample, using different measures of thickness of the tails and aggregate profits. Additionally, the results also obtain when the variables are constructed using firm-level data from Orbis.

Finally, to assess the economic relevance of the mechanism I turn to a quantitative application, focusing on the external imbalances between Germany and the rest of Europe. There are several reasons to focus on Germany. First, Germany is responsible for 60% of the foreign assets accumulated by lender economies as of 2019. Thus, European imbalances are, to a large extent, German imbalances. Second, German producers stand out among European peers: in my sample, Germany ranks second in terms of its aggregate profit share and first in terms of the prevalence of ‘superstar’ firms. Finally, German firms are also ‘closely held’, with relatively underdeveloped equity markets and most firms in private ownership. Thus, German profits largely accrue to German capitalist households.

In order to take the model to the data I extend it in several of ways. The quantitative model features multiple sectors and costly trade. Claims to future profit streams are transferable, giving rise to a second type of asset – firm equity – in addition to capital. Firm productivity draws follow a Pareto distribution. Finally, I model the household side following [Straub \(2018\)](#). Non-homothetic asset demand now arises endogenously, as richer households value bequests more and prefer to postpone their spending until later in life. As a result, richer capitalist households hold more assets as compared to the workers.

The quantitative model yields three results. First, in the simulation, Germany features a higher aggregate profit share than the Rest-of-Europe aggregate, in line with the data. Notably, profit shares are not a targeted moment, and instead arise endogenously due to the heterogeneous firm productivity distributions disciplined by the data. Second, under financial liberalization, the model generates German net foreign asset holdings of 14% of GDP – a quarter of that in the data. Thus, the profit mechanism contributes importantly to our understanding of European imbalances. Finally, I study the role of different sources of

heterogeneity in the model. I find that heterogeneous tail parameters in firm productivity distributions alone can generate 70% of the imbalances in the model. Since this parameter determines the prevalence of firms with extreme productivity draws, I argue that the prevalence of such ‘superstar’ firms is the key driver of external imbalances in the model.

I conclude the paper by asking how the United States fits the story by revisiting the imbalances between the United States and rest of world. I show that in a stylized simulation where the United States features higher concentration, higher aggregate profit share, and a better capacity to generate financial assets, it holds net foreign debt. The exercise highlights the interaction between the relatively poorly developed financial markets in Europe and the profits mechanism in driving the European imbalances.

Literature review. This paper forms part of a literature on ‘global imbalances’, typically understood as arising from asset market asymmetries in different parts of the world.² For example, [Caballero et al. \(2008\)](#) focus on lower supply of assets in the lender economies due to the lack of a developed financial system. [Mendoza et al. \(2009\)](#) and [Ferrero \(2010\)](#), instead, explain the imbalances as caused by differences in asset demand, e.g. due to the amount of idiosyncratic risk faced by households or heterogeneous demographic pressures. In this paper, I propose a different source of asset market asymmetry: the share of pure profits in the economy. In a related paper, [Atkeson et al. \(2022\)](#) link external imbalances to aggregate profits, arguing that the valuation effects of an unexpected change in firm profitability can explain the deterioration of the US net foreign account. In comparison, I focus on the steady state determinants of external imbalances where no valuation effects occur. Additionally, while the asset supply operates similarly in the two papers – a more profitable economy features a lower capital to GDP ratio but generates more financial assets – I argue that profits affect asset markets via the asset demand as well as the asset supply channels, and show that an active asset demand channel is necessary to match European imbalances quantitatively.

²In this paper, I mainly use ‘external imbalances’ as opposed to ‘global imbalances’ as the latter tends to specifically refer to external positions of the US and China.

A closely related strand of literature studies the long term decline in global interest rates that has occurred in recent decades. A number of recent contributions have linked the declining capital share and interest rate with a trend of rising market power (De Loecker et al. (2020), Liu et al. (2022)). The mechanism in my paper, whereby higher profit shares suppress asset supply, functions similarly. Mian et al. (2021), instead, link declining interest rates to growing inequality. The mechanism in their paper relies on heterogeneities in saving behavior of different population groups. In my paper a similar mechanism links profits, through the higher propensity to save by the recipients, to higher asset demand. In my paper, the two mechanisms are brought into motion through trade in imperfectly competitive markets and are the driver of countries' external positions.

The central element of my model is heterogeneous profit shares which arise due to oligopolistic competition. Recently, there has been a resurgence in the use of oligopolistic competition models to study the behavior of markups in both macroeconomics (Edmond et al. (2023), Burstein et al. (2020)) and the trade literature (Bernard et al. (2003), Atkeson and Burstein (2008), Gaubert and Itskhoki (2021)). That the aggregate profit share is shaped by heterogeneous markups at the firm level is a standard result in this literature. In this paper, I sharpen this result, arguing that it is the prevalence of extremely productive 'superstar' firms that determines the level of aggregate profits.

Finally, a number of papers have focused on the nexus between external imbalances, trade and TFP. Reis (2013) and Gopinath et al. (2017) argue that international borrowing can lead to declines in TFP as capital inflows increase the misallocation of capital. Notably, both papers make Europe their case study. Inasmuch as both papers take the existence of external imbalances as given, the present paper can be viewed as taking one step back and asking what can explain their origin. The closest paper is de Ferra (2021), which also focuses on European imbalances. In it, imbalances emerge due to implicit subsidies to holdings of assets generated in Southern economies. The mechanism proposed in this paper is distinct, so the two contributions can be viewed as complementary.

The rest of the paper is organized as follows. Section 2 presents a stylized model where oligopolistic competition between firms generates external imbalances, and maps the predictions of the model to the empirical patterns in Europe. In Section 3, I present the quantitative version of the model. Section 4 outlines the calibration and the counterfactuals, whereas Section 5 addresses the main question of the paper: how do the profits affect external imbalances between Germany and rest of Europe? Finally, Section 6 concludes.

2 Stylized model

In this section, I outline the setup of the two-country model where firms compete oligopolistically, discuss the determinants of the aggregate profit share, and characterize the steady state of the model under financial autarky and financial liberalization.

2.1 Model Setup

The model features two countries, home and foreign. Foreign variables are marked by asterisks. There are N firms in each economy producing non-overlapping, heterogeneous varieties, and a final good producer that combines the varieties into a final good. There are two types of households in each economy: workers and capitalists. Home and foreign are symmetric, with the exception of firm productivities. I derive the optimality conditions for the domestic firms and households, suppressing the corresponding conditions for the foreign economy for ease of exposition. I then characterize the equilibrium of the model. The model is kept intentionally simple to aid tractability, several extensions are added in Section 3.

Firms in the common market. Domestic firms are indexed by $n \in \{1, \dots, N\} = \mathcal{N}$. They are heterogeneous in their productivity z_n and produce using a Cobb-Douglas production function:

$$q_n = z_n k_n^\alpha l_n^{1-\alpha}, \quad \alpha \in [0, 1).$$

Intermediate goods can be traded costlessly. The intermediate goods are combined into a final good by the final good producer using CES technology:

$$Q = \left[\sum_{n \in \mathcal{M}} q_n^{\frac{\sigma-1}{\sigma}} + \sum_{n^* \in \mathcal{M}^*} q_{n^*}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution, and where \mathcal{M} , \mathcal{M}^* are the sets of firms that operate in equilibrium. The final good is non-tradable and its market is perfectly competitive.

The finite number of firms results in an oligopolistic competition structure in the intermediate goods market. I assume that firms compete on quantity, à la Cournot. [Atkeson and Burstein \(2008\)](#) show that in this case, firm prices and market shares are jointly determined by the firm profit maximization conditions and the final good producer demand for intermediate goods, such that firm n 's price P_n is

$$P_n = \frac{\sigma}{\sigma-1} \frac{c_n}{1-s_n}, \quad \text{where} \quad c_n = \left(\frac{w}{1-\alpha} \right)^{1-\alpha} \left(\frac{r}{\alpha} \right)^{\alpha} \frac{1}{z_n} \quad (1)$$

is the marginal cost of production of firm n , w is the wage, r is the rental cost of capital, and s_n is firm n 's sales share in the common market:

$$s_n = \frac{y_n}{\sum_{n \in \mathcal{M}} y_n + \sum_{n^* \in \mathcal{M}^*} y_{n^*}} = \frac{P_n^{1-\sigma}}{\sum_{n \in \mathcal{M}} P_n^{1-\sigma} + \sum_{n^* \in \mathcal{M}^*} P_{n^*}^{1-\sigma}}. \quad (2)$$

I assume that there are no fixed costs of operation. This means that all N firms at home and abroad operate: $\mathcal{M} = \mathcal{M}^* = \mathcal{N}$. Variety output, y_i , is its share of the global output:

$$y_n = s_n (Y + Y^*), \quad (3)$$

where Y and Y^* are revenues of home and foreign final goods producers respectively.

Firm profit share, i.e. the share of profits in its revenue, is linear in firm's market share:

$$\pi_n = \frac{\Pi_n}{y_n} = \frac{y_n - c_n q_n}{y_n} = 1 - \frac{c_n}{P_n} = \frac{1}{\sigma} + \frac{\sigma - 1}{\sigma} s_n. \quad (4)$$

Firm-level factor demand comes from firm optimality conditions:

$$\begin{aligned} rk_n &= \alpha c_n q_n = \alpha(1 - \pi_n)y_n, \\ wl_n &= (1 - \alpha)c_n q_n = (1 - \alpha)(1 - \pi_n)y_n. \end{aligned}$$

Foreign firms operate symmetrically, yielding optimal $\{s_n^*, \pi_n^*, k_n^*, l_n^*\}$ for each firm.

Households. There are two types of households: workers and capitalists, with measures $(1 - \mu)L$ and μL respectively. Workers supply labor inelastically and earn wages w . Capitalists work and earn wages, but also receive the profits of domestic firms. Firm ownership is pooled across the capitalist households, so each receives $\sum_n \pi_n y_n / (\mu L)$.³

Household utility increases in consumption of the final good.⁴ The budget constraint for workers and capitalists is as follows:

$$\begin{aligned} C^w + S^w &= ra^w + w, \\ C^c + S^c &= ra^c + w + \frac{\sum_n \pi_n y_n}{\mu L}, \end{aligned}$$

where C^i is per-capita consumption by each of the household types, S^i represents savings in a given period, and a^i are the assets held by each household. Capital does not depreciate, so the rental rate of capital and the return on asset holdings is the same, r . I focus on the steady state, so time subscripts are suppressed and $S^i = 0$ for both household types.

For the purposes of the stylized model, I characterize households by an asset demand

³I assume that firm ownership is not transferable. This would be the case if the claims to future profits are not contractible. I make this stringent assumption to match qualitatively the relatively low market capitalization in Europe, and relax it in the quantitative version of the model.

⁴This ensures that household budget constraints hold with equality.

that is proportional to their per-capita non-financial income by a factor ζ^i :

$$a^w = \zeta^w w, \quad a^c = \zeta^c \left(w + \frac{\sum_n \pi_n y_n}{\mu L} \right).$$

Per-capita asset holdings in foreign are symmetric and are denoted by a_w^* , a_c^* . I assume that $\zeta^c > \zeta^w$. This setup, in a reduced form, captures the idea that asset demand is non-homothetic: the richer, in per-capita terms, capitalist households demand more assets as a share of their non-financial income.⁵ Since non-homothetic asset demand does not typically give rise to closed form policy functions, I defer the specification with endogenous asset demand to the quantitative model of Section 3.

Markets clearing. The model is closed by pricing the factors of production: capital and labor. Aggregating across firms, home capital demand satisfies:

$$rK = \sum_{n \in \mathcal{N}} \alpha(1 - \pi_n) y_n = \alpha(1 - \pi) Y, \quad \text{where } \pi = \frac{\sum_{n \in \mathcal{N}} \pi_n y_n}{\sum_{n \in \mathcal{N}} y_n} \quad (5)$$

is the aggregate profit share in the economy.

Asset demand A can be obtained by summing up individual asset demands of domestic workers and capitalists:

$$A = \mu L a_c + (1 - \mu) L a_w = \mu L \zeta^c \left(w + \frac{\sum_n \pi_n y_n}{\mu L} \right) + (1 - \mu) L \zeta^w w. \quad (6)$$

If the two economies are in financial autarky, then capital markets clear domestically (Case (a)). If capital can flow freely across borders, then capital markets clear globally (Case (b)) and the interest rates are equalized at home and abroad at some global level r^G :

⁵This idea has a long history in economics, dating back to Fisher (1930), and has been supported empirically (Dynan et al., 2004; Straub, 2018; Fagereng et al., 2019). There are many possible reasons for the asset demand to feature non-homotheticity. De Nardi (2004) models it as arising due to the households treating bequests as a luxury good, while Straub (2018) finds that each of non-linear social security system, non-homothetic preferences for bequests and non-homothetic preferences for the distribution of consumption across periods play a role in explaining the disproportionate asset holdings of the rich.

$$K = A, \quad (7a)$$

$$K + K^* = A + A^*, \quad r = r^* = r^G. \quad (7b)$$

Wage ensures that the labor supplied by the households satisfies the labor demand:

$$wL = \sum_{n \in \mathcal{N}} (1 - \alpha)(1 - \pi_n)y_n = (1 - \alpha)(1 - \pi)Y. \quad (8)$$

Home output is the sum of the sales of domestic firms:

$$Y = \sum_{n \in \mathcal{N}} y_n. \quad (9)$$

The foreign economy is symmetric and yields a set of outcomes $\{K^*, A^*, r^*, w^*, Y^*\}$. Finally, I normalize the global expenditure to one:

$$Y + Y^* = 1. \quad (10)$$

Definition 1: (Steady state equilibrium) An equilibrium is a sequence of $\{s_i, y_i, \pi_i\}_N, \{s_i^*, y_i^*, \pi_i^*\}_{N^*}, r, r^*, w, w^*, Y, Y^*$ such that (i) firm's market share, output, and profit share satisfy the firm's optimal pricing conditions and the final good producer's demand, (1)–(4), (ii) the interest rates equalize the aggregate capital demand given in (5) and asset demand given in (6) subject to the capital market clearing condition (7a) in the case of autarky and (7b) in the case of financial liberalization, (iii) wages satisfy the respective labor market clearing conditions (8), (iv) aggregate outputs satisfy (9), and (v) normalization holds (10). The goods markets clear by Walras' law.

2.2 Aggregate Profit Share and ‘Superstar’ Firms

From firm-level to country-level profit share. In the simple model of oligopolistic competition presented above, the source of higher profit share of a firm, π_i , is its cost advantage in the form of a relatively low unit production cost c_n . This means that the firm is able to offer a lower price than that of its competitors, attracting a larger share of the market s_n :

$$\pi_i = \frac{1}{\sigma} + \frac{\sigma - 1}{\sigma} s_i.$$

Consider now the aggregate counterparts to firm-level profit share and the market share:

$$\pi = \sum_{n \in \mathcal{N}} d_i \pi_i, \quad s = \sum_{n \in \mathcal{N}} d_i s_i,$$

where $d_i = y_i / \sum_{n \in \mathcal{N}} y_i$ is the domestic sales share of firm i , π is the aggregate profit share, and s is the weighted average share of domestic firms in the common market. As in the case of the per-firm variables, the aggregates also satisfy:

$$\pi = \frac{1}{\sigma} + \frac{\sigma - 1}{\sigma} s.$$

In other words, economies with bigger firms also feature higher aggregate profit shares. Finally, note that the average market share s can be rewritten as follows:

$$s = \frac{\sum_{n \in \mathcal{N}} s_i^2}{\eta}, \quad \text{where } \eta = \frac{Y}{Y + Y^*}.$$

This alternative presentation aids the economic interpretation of the average share of domestic firms in the common market, s . An economy with a small population will necessarily have a limited presence in the world markets. However, it may still be an important player internationally. Thus, an alternative interpretation of a high s is that this is an economy that produces many ‘superstar’ firms (firms with $s_i^2 \gg 0$), given its size η .

Fundamental determinants of the aggregate profit share. Both the average share in the common market and the aggregate profit share are endogenous objects in the model. Meanwhile, the fundamental source of heterogeneity among countries in the model is the differing productivity distribution of their firms. Thus, in order to understand what, at a fundamental level, makes for a high aggregate profit share economy, one needs to understand how it is affected by the firm-level productivities.

Let firms be indexed such that the productivities of domestic firms decline in n : $z_1 \geq z_2 \geq \dots \geq z_N$. The derivative of the aggregate profit share with respect to the firm i 's productivity is proportional to the difference between the elasticities of wage and output per worker, $y = Y/L$, both with respect to the firm i 's productivity:

$$\frac{d\pi}{dz_i} = \frac{z_i}{1 - \alpha} \left(\frac{dy/y}{dz_i/z_i} - \frac{dw/w}{dz_i/z_i} \right).$$

Moreover, both elasticities are positive: a higher productivity of firm i simultaneously increases output per worker and pushes up the workers' wage. Thus, the effect of firm i 's productivity on the aggregate profit share is ambiguous: higher productivity of any one firm does not necessarily make an economy more profitable. In Appendix A.1, I show that the sign of the derivative depends on the firm's ranking in the domestic economy.

Proposition 1: An increase in productivity of the *most productive* firm increases the aggregate profit share:

$$\frac{dw/w}{dz_1/z_1} \leq \frac{dy/y}{dz_1/z_1}, \quad \text{and thus} \quad \frac{d\pi}{dz_1} > 0.$$

If $2s_N \leq s$, i.e. if the firm productivity distribution is sufficiently dispersed, then an increase in productivity of the *least productive* firm decreases the aggregate profit share:

$$\frac{dw/w}{dz_N/z_N} \geq \frac{dy/y}{dz_N/z_N}, \quad \text{and thus} \quad \frac{d\pi}{dz_N} \leq 0.$$

In other words, having more extreme draws for the most productive – ‘superstar’ – firm

results in a higher aggregate profit share compared to an economy with an otherwise identical firm productivity distribution.

Note that this relationship relies crucially on oligopolistic competition where the largest firms earn higher rents. The most productive firm in the economy has both the largest market share and the labour share among the domestic firms. However, under oligopolistic competition, the sales share of the largest firm is larger than its labor share: $d_1 > l_1$. Not only are firms restricting their supply (and therefore inputs) to earn rents, but the most productive firms do so to a greater extent. As a result, a ‘superstar’ firm that expands does not lift wages much. The oligopolistic behavior of such firms makes for relatively low wages given the country’s productivity, thereby increasing the aggregate profit share.

2.3 Aggregate Profit Share and Capital Flows

I now turn to discussing how aggregate profits affect the asset markets.

Steady state under financial autarky. Consider the case of autarky first. The autarkic interest rate r_a clears the domestic asset market:

$$K = \frac{\alpha}{r_a} Y(1 - \pi) = \zeta_c(\mu(1 - \alpha)(1 - \pi) + \pi)Y + \zeta_w(1 - \mu)(1 - \alpha)(1 - \pi)Y = A,$$

where the right-hand side has the income of the workers and capitalists expressed as a multiple of GDP. Note that both the asset supply and asset demand depend on the aggregate profit share. Consider each in turn.

Aggregate asset supply as a share of GDP declines in the aggregate profit share:

$$\frac{K}{Y} = \frac{\alpha}{r_a}(1 - \pi), \quad \frac{d\frac{K}{Y}}{d\pi} < 0.$$

Firms hire capital until its marginal revenue product equals the rental cost. Under imperfect competition, firms internalize the fact that expanding supply reduces their prices. As

firms restrict their supply they hire less capital, compared to the competitive benchmark.⁶ As a result, less capital is available as a store of value, *as a share of GDP*. Note further that this result holds as a matter of accounting. Firms that earn large profits have a larger share of their revenue construed by profits, as opposed to value added. A lower value added, for a given interest rate, requires fewer inputs. Thus, *ceteris paribus*, high profit shares necessarily imply a smaller capital to sales ratio.

Aggregate asset holdings as a share of GDP increase in the aggregate profit share:

$$\frac{A}{Y} = \zeta_c(\mu(1 - \alpha)(1 - \pi) + \pi) + \zeta_w(1 - \mu)(1 - \alpha)(1 - \pi), \quad \frac{d\frac{A}{Y}}{d\pi} > 0. \quad (11)$$

A higher profit share redistributes income towards the capitalists and, therefore, towards the demographic with a higher demand for assets, raising the aggregate.

Higher aggregate profit share simultaneously suppresses the asset supply and increases the asset demand. Both of these effects bring forth a decline in the autarkic interest rate:

$$r_a = \frac{\alpha(1 - \pi)}{\zeta_c(\mu(1 - \alpha)(1 - \pi) + \pi) + \zeta_w(1 - \mu)(1 - \alpha)(1 - \pi)}, \quad \frac{dr_a}{d\pi} < 0. \quad (12)$$

Since all the parameters other than firm productivities are symmetric between home and foreign, the following proposition holds.

Proposition 2: (Steady state under financial autarky) If home features a higher aggregate profit share than foreign, then in the steady state under financial autarky home's autarkic interest rate is lower than that in the foreign economy. See Appendix A.2 for the proof.

Steady state under financial integration. If capital is allowed to flow freely, the home and foreign interest rates will be equalized at some global level r^G and the global capital

⁶Here, restricting inputs to restrict supply should be read as firms using less inputs *per unit of sales*. This is immediate from firm's FOCs and arises in standard models where firms exert market power: $k_n/y_n = \alpha(1 - \pi_n)/r < \alpha/r = k_n^c/y_n^c$, where k_n^c/y_n^c is the capital per unit of sales under perfect competition.

market will clear subject to (7b). The global interest rate will be as follows:

$$r^G = \frac{\alpha(1 - \pi^G)}{\zeta_c(\mu(1 - \alpha)(1 - \pi^G) + \pi^G) + \zeta_w(1 - \mu)(1 - \alpha)(1 - \pi^G)}, \quad (13)$$

where $\pi^G = \eta\pi + (1 - \eta)\pi^*$ is a weighted average of the home and foreign profit shares.

Under financial integration, home asset demand need not be satisfied by domestic assets. Countries can both lend and borrow, taking up positive and negative net foreign asset positions. In Appendix A.3 I show that home net foreign assets as a share of GDP will be a function of the aggregate profit share differential between home and abroad:

$$\frac{NFA}{Y} = \frac{A - K}{Y} = \zeta_c(1 - \eta) \frac{(\pi - \pi^*)}{1 - \pi^G}. \quad (14)$$

Proposition 3: (Steady state under financial integration) If home features a higher aggregate profit share than foreign, home's net foreign asset position is positive in the steady state under financial integration.

2.4 Empirical Evidence

In this section, I discuss external imbalances, aggregate profit shares and firm size distributions in Europe, and link these to the predictions of the model. I begin by briefly outlining the data. See Appendix B.1 for the data sources and variable construction.

Data and Measurement. Estimating firm size distributions for a large set of economies hinges on collecting harmonized and representative firm-level data in a cross-country setting. Unfortunately, such data is rarely available. As an alternative, I follow Chen (2022) in leveraging employment by sector and firm size bin from the OECD's Structural Business Statistics (SBS) dataset to construct an index of the thickness of the right tail:

$$tail = \log \frac{\tilde{F}(T_L)}{\tilde{F}(T_S)} / \log \frac{T_L}{T_S}, \quad (15)$$

where T_L is the firm size threshold for large firms, T_S is the threshold for small firms, and $\tilde{F}(x)$ is the counter-cumulative distribution function of firm employment. If the underlying firm size distribution is Pareto, then *tail* estimates the shape parameter of the underlying distribution. For other distributions, *tail* measures the prevalence of very large firms in the right segment of the distribution. I set T_L to 250+ workers (largest bin) and T_S to 10+ workers, and compute \overline{tail} as the average tail index of all two-digit non-financial corporate sectors of the economy. The dataset covers years from 2005 to 2017.⁷

I measure profits in two complementary ways.⁸ The main method, following [Barkai \(2020\)](#), computes profits as the difference between the gross operating surplus of the non-financial corporate sector (revenue less the wage bill) and the imputed capital costs. I construct the latter by multiplying the nominal capital stocks of the non-financial corporate sector by the sum of the bank interest rate on corporate loans and the capital depreciation rate. The second measure of profits is the entrepreneurial income of the non-financial corporate sector. Entrepreneurial income constitutes the national accounts equivalent of the profit or loss in business accounting and is calculated by deducting from operating surplus any interest and rent payable and adding property incomes receivable. The first measure recuperates pure profits but relies on an imputed measure of capital costs. The second uses the recorded capital payments, but, as with any accounting profit, contains additionally the return on owned capital.⁹ The first measure covers the period from 2003 to 2019 while the second one covers the period from 1995 to 2019.¹⁰

European Imbalances. Since the 1990s Europe underwent a period of financial liberalization, involving wide ranging legal and regulatory harmonization in the financial markets and, ultimately, adoption of the Euro by twelve economies in 1999-2001 (see [Kalemli-](#)

⁷Results are robust to setting T_S to 50+ workers.

⁸I do not construct profits by computing aggregate markups as obtaining markups in levels hinges on using representative cross-country firm-level data on prices and quantities (see [De Ridder et al. \(2022\)](#)).

⁹In the data, capital costs share is orthogonal to the aggregate profit share computed using the main method. If additionally the share of capital that is owned is orthogonal to the aggregate profit share, then entrepreneurial income comprises a valid instrument for measuring aggregate profit.

¹⁰I end my analysis in 2019, just before the COVID-19 pandemic shock.

Ozcan et al. (2010) for a detailed discussion). The increased financial integration between the European states has led to a significant increase in intra-European cross-border financial linkages. The same period witnessed the development of large European imbalances.

I define net lender economies as those that held positive net foreign asset positions in 2019: Netherlands, Denmark, Germany, Belgium, Austria, Sweden and Finland. Each of these ran average current account surpluses of over 1% of GDP between 1995 and 2019. I refer to the rest of economies – Spain, France, Greece, Italy and Portugal – as net borrowers.¹¹ Figure 1 shows that starting from an essentially identical position, the average net foreign assets of the two groups have diverged.¹² Net borrower economies have accumulated foreign debt up until the crises of the late 2000s brought international capital flows to a halt. Since then, the external position of the net borrower economies has remained roughly stable. Net lender economies, in turn, have seen their net foreign assets rise from 2001 onwards. By 2019, the gap between the external positions of the two groups has reached over 100% of GDP. Notably, as a result of these offsetting dynamics the external position of Europe as a whole has been roughly balanced over the period.¹³

‘Superstar’ firms, aggregate profit shares, and external imbalances. Figure 2 presents the main stylized fact motivating the mechanism. In Panel (a) I plot countries’ average tail indices against their net foreign asset positions. In Panel (b), I plot the aggregate profits against the net foreign assets (both as a share of GDP). Each dot represents a country-year observation. I find that, in Europe, both higher prevalence of ‘superstar’ firms and high aggregate profit shares were associated with higher net foreign asset holdings. Motivated by this evidence, I proceed to test the predictions of the model more formally.

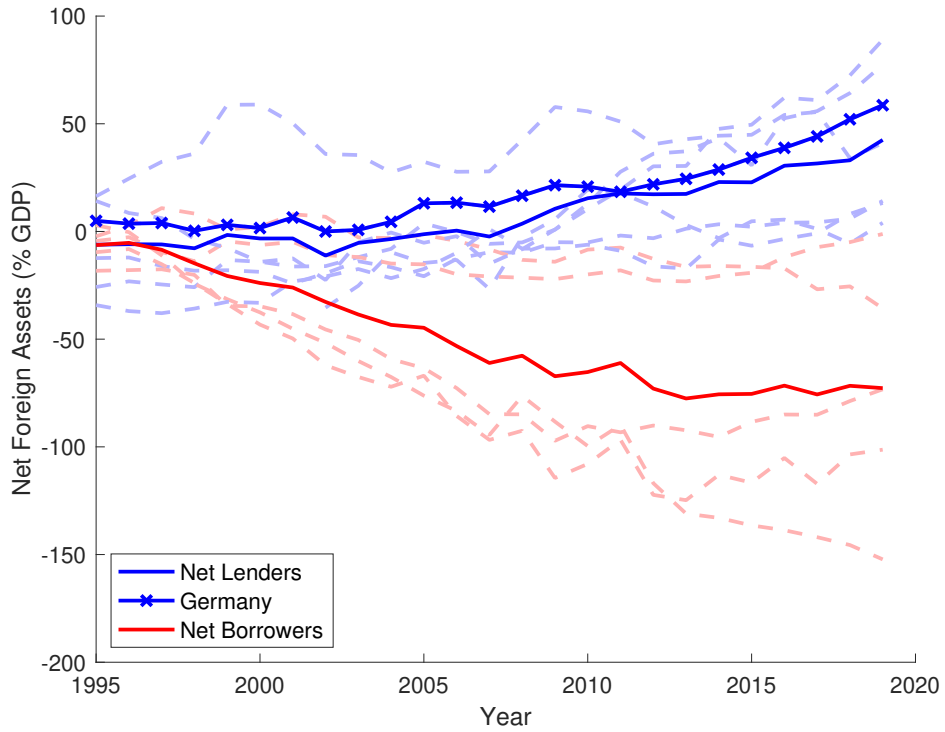
Estimation. Since the model describes the steady state and links slow-moving objects such as the firm size distribution and the net stock of foreign assets, I test the predictions

¹¹I exclude from analysis the former Eastern bloc economies, financial center economies (Malta, Ireland, Cyprus, Luxembourg), and Norway, where net foreign assets are dominated by the sovereign wealth fund.

¹²I obtain net foreign assets from External Wealth of Nations database (Lane and Milesi-Ferretti, 2018).

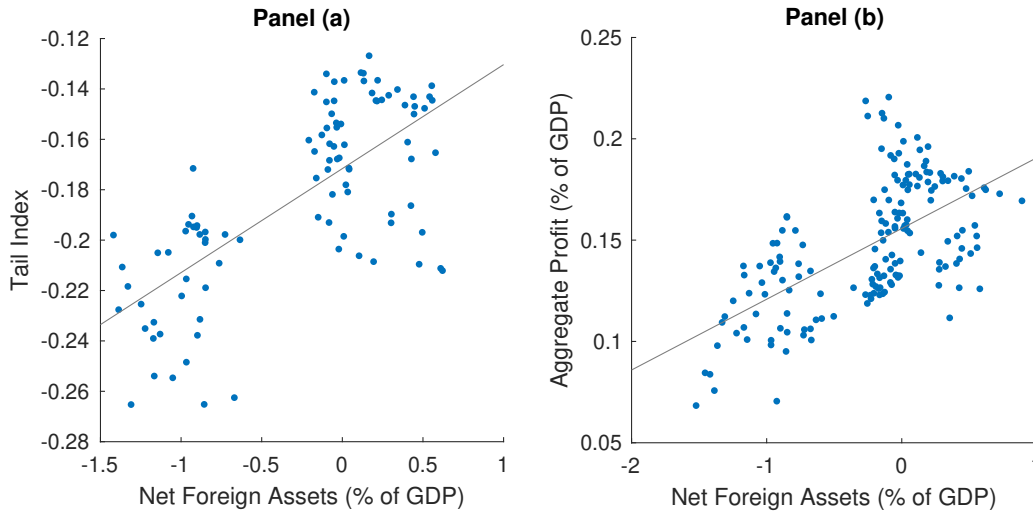
¹³Starting at -3% in 1995, it decreased temporarily, returning back to -3% by 2015, and finally drifting into mildly positive territory in recent years.

Figure 1: External Imbalances in Europe



Note: Net lenders: Netherlands, Denmark, Germany, Belgium, Austria, Sweden, Finland. Net borrowers: Spain, France, Greece, Italy, Portugal. Sources: External Wealth of Nations Database (EWN).

Figure 2: Profits, 'Superstar' Firms and External Imbalances in Europe



Note: Panel (a): Net foreign assets as a share of GDP against the average tail index, 2005-2017. Panel (b): Net foreign assets as a share of GDP against the aggregate profit share, 2003-2019.

of the model using pooled panel regressions. To control for global shocks that might drive endogenous variables at the business cycle frequency, I add year fixed effects in all specifications. I cluster standard errors at a country level, thus exploiting the cross-sectional variation in the data. Finally, as my model abstracts from the long-run determinants of countries' external positions other than profits, I additionally run specifications that control for other potential drivers of external positions.¹⁴

Results. Column 1 in Table 1 shows that economies with a higher prevalence of superstar firms enjoy higher aggregate profit shares: a one standard deviation increase in \overline{tail} (0.06) increases the aggregate profit share by 2.4 percentage points, consistent with firms engaging in oligopolistic competition. Columns 2 and 4 speak to the main prediction of the model: the thickness of the tails of firm size distributions and aggregate profit shares are both associated with higher net foreign asset positions. In terms of magnitude, a one standard deviation increase in \overline{tail} leads to 45 percentage points increase in net foreign assets as a share of GDP, whereas a one standard deviation increase in the aggregate profit share (3.1 p.p.) leads to 33 percentage points increase in the net foreign asset position. Finally, Columns 3 and 5 show that these results are robust to controlling for other potential drivers of external imbalances.

Robustness. In Appendix B.2 I present several robustness checks. First, I show that the results are not specific to Western Europe: all the relationships hold for a full sample that includes Eastern European and non-European economies. Second, the results are robust to using different measures of the thickness of the tail and aggregate profit shares. Third, results are robust to excluding the years of the Great Recession and the European debt crisis. Finally, I re-do the analysis using the Orbis Historical dataset by BvD Electronic Publishing. I find that the predictions of the model hold in this alternative dataset.

¹⁴I add initial capital stock (% GDP, measured in 2000) to control for foreign debt accumulation predicted by the neoclassical model, the Rule of law index from World Bank's Worldwide Governance Indicators which measures the quality of contract enforcement and property rights to control for the shortage of stores of value associated with a poorly developed financial system (Caballero et al., 2008), and population growth and old-age to working-age ratio to control for demographic drivers of external positions (Ferreiro, 2010).

Table 1: Regression Results

	Agg. Profit (% GDP)		Net Foreign Assets (% GDP)		
	(1)	(2)	(3)	(4)	(5)
\overline{tail}	0.415*** (0.057)	7.932*** (1.328)	5.946*** (1.428)		
pr				10.653*** (3.116)	8.989** (3.222)
FE	Y	Y	Y	Y	Y
Clustering	C	C	C	C	C
Controls			✓		✓
Observations	84	98	98	168	168
R ²	0.511	0.548	0.734	0.376	0.574

Note: \overline{tail} is the country-year average of the tail index described in equation (15), pr is the country-year aggregate profit share. C and Y stand for county and year. Controls include the initial capital stock (% GDP, measured in 2000), the Rule of law index from World Bank's Worldwide Governance Indicators, population growth and old-age to working-age ratio. * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

3 Quantitative Model

The stylized model has abstracted from a number of dimensions that are relevant for studying the quantitative importance of the profit mechanism. In this section, I show how the model of Section 2 can be extended to allow for multiple countries, sectors, costly trade, trade in financial assets, and endogenous asset demand.

3.1 From Stylized to Quantitative Model

The fully fledged model differs from the stylized model as follows. First, I allow for $K \geq 1$ sectors. Multiple sectors enter through a higher level aggregation in the final goods production function, which now becomes Cobb-Douglas in the sectoral goods with

weights γ_{ik} , nested with CES at a variety level:

$$Q_i = \prod_K Q_{ik}^{\gamma_{ik}}, \quad \text{where} \quad Q_{ik} = \left[\sum_{j \in \mathcal{I}} \sum_{n \in \mathcal{M}_{ijk}} q_{ijkn}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad \sum_K \gamma_{ik} = 1.$$

Economies are now indexed by $i \in \{1, \dots, I\} = \mathcal{I}$. While in the quantitative application I will focus on two economies, the model admits $I \geq 2$, while economizing on notation.

Second, I allow for costly trade. This extension is important as trade costs protect domestic firms from competition, and thus have a first order effect on aggregate profits. While costless trade yields one common market for each sector, costly trade means that there are as many (sector-level) markets as there are economies. Firms may choose how much to export in any given market, and what markup to charge on their exports, independently of their domestic sales considerations. Thus, firm sales shares and profit shares are now determined for each of the markets the firm serves, with index j denoting the market: s_{jikn}, π_{jikn} . I introduce trade frictions as iceberg costs, applying as a percentage over the marginal costs. Thus, costs of production are now market specific:

$$c_{jikn} = \left(\frac{w_i}{1 - \alpha} \right)^{1-\alpha} \left(\frac{r_i}{\alpha} \right)^{\alpha} \frac{d_{jik}}{z_{in}},$$

where $d_{iik} = 1$ for all $i \in \mathcal{I}$.

Third, I specify a parametric distribution from which firms make their productivity draws. I assume firms draw productivities from a Pareto distribution, with CDF

$$G_{ik}(z) = 1 - \left(\frac{z_{ik}}{z} \right)^{\theta_{ik}},$$

where z_{ik} and θ_{ik} are the country-sector specific scale and tail parameters of the Pareto distribution.

In the stylized model I assumed that firm ownership is non-transferable, that is, the future profit streams can not be capitalized into traded financial claims. For the quantitative

version of the model, I relax this assumption. Instead, I introduce limited pledgeability of such flows, parameterized by parameter λ (I follow [Caballero et al. \(2008\)](#), who restrict $\lambda < 1$ since agents can dilute and divert part of the profits). Limited pledgeability is important to match quantitatively the relatively low stock market capitalization in Europe. This assumption changes the asset supply in the model, which now is a sum of domestically held capital K_i and the tradable share of the value of firms in the economy, λF , where $F = \sum_t^\infty (1+r)^{-t} \Pi_t$. Furthermore, the stream of profits from the non-tradable portion of the firms that accrue to the capitalists is now reduced, at $(1-\lambda)\Pi$.

Finally, I now introduce a fully-fledged asset demand for the two types of households. I borrow the setup from [Straub \(2018\)](#), stripping away the individual income- and date-of-death uncertainty. Households are born and live for T periods in an overlapping generations manner. The birth rate is $1/T$, so the size of the population remains constant. The two groups of population, workers and capitalists, represent dynasties with no mobility between the types: workers give birth to workers and capitalists to capitalists. Agents are economically inactive until the age of $t_1 = (T+1)/3$, at which point they enter the labor force. At age t_2 agents leave the workforce and stay retired until the age of death T . At $(T+1)/3$, each agent gives birth to one child. Labor is taxed at τ^{lab} , with the tax receipts paid out as pension transfers T^{soc} to the concurrently living retired. The only difference in the income stream between the workers and capitalists is that capitalists own the non-tradable share of firms in the economy, and thus, as a group, receive share $1-\lambda$ of the aggregate profits. I take no stance regarding the distribution of profits across age, and simply assume that each cohort receives an equal share.¹⁵ Upon death, the stake in the firms is passed to the youngest economically active agent in the dynasty. In short, the non-financial income of the two types of households is as follows:

¹⁵This ensures that these profit streams do not generate a smoothing motive of their own.

$$y_s^w = \begin{cases} (1 - \tau^{lab})w & \text{if } t_1 \leq s \leq t_2, \\ T^{soc}w & \text{if } t_2 < s \leq T, \end{cases} \quad y_s^c = \begin{cases} (1 - \tau^{lab})w + \frac{\Pi(1 - \lambda)}{\mu L(T - t_1)} & \text{if } t_1 \leq s \leq t_2, \\ T^{soc}w + \frac{\Pi(1 - \lambda)}{\mu L(T - t_1)} & \text{if } t_2 < s \leq T. \end{cases}$$

The social security budget is balanced, so $(T - t_2)T^{soc} = (t_2 - t_1)\tau^{lab}w$. The budget constraint is standard:

$$c_t^i + a_t^i = y_t^i + (1 + r_t)a_{t-1}^i, \quad \text{where } i \in \{w, c\}.$$

Agents receive inheritance from their grandparent at t_1 , so the asset holdings at the start of economic life are the assets held at death by the grandparent, $a_{t_1}^i = a_T^i$.

Each agent has a utility function that depends on per-period consumption and on the bequest left at the time of death:

$$U = \sum_{s=T/3}^T \beta^s u_s(c_s^i) + U_a(a_T^i).$$

Following [Straub \(2018\)](#), I pick

$$u_s(c) = \frac{(c/o)^{1-\nu_s}}{1 - \nu_s}, \quad \text{where } \nu_s > 0, o > 0,$$

where ν_s is an age-dependent parameter that governs the income elasticity of consumption over the life-cycle, and o is a normalization parameter. In turn,

$$U_a(a) = k \frac{((a + \underline{a})/o)^{1-\nu_T}}{1 - \nu_T}, \quad \text{where } \sigma > 0, k > 0, \underline{a} > 0.$$

This setup generates two sources of non-homotheticity in asset holdings. First, the intercept in the bequest part of the utility function ensures that bequeathing is a luxury: richer agents save more to leave a larger inheritance for their grandchild. There is extensive evidence that bequests as a share of income do indeed increase as individuals get richer

(Carroll, 2000; Dynan et al., 2004). Second, I follow Straub (2018) in parameterizing ν_s to decline in age, with $\nu_{s+1}/\nu_s = \sigma^{slope} < 1$. This generates a higher late-life expenditure amongst the richer agents in the economy, thus encouraging them to accumulate assets for late-life consumption. Such expenditures can be thought of as, e.g., college fees for the children, medical procedures or vacations during retirement, all of which are more prevalent among the higher-income households. The rest of the model remains unchanged.¹⁶

The endogenous variables in the quantitative model are $\{s_{jkn}, P_{jkn}, w_i, r_i, Y_i\}$ for countries $i, j \in \mathcal{I}$, sectors $k \in \mathcal{K}$ and firms $n \in N$, and a vector of consumption and asset holdings for each type of agent and each age: $\{c_s^w, c_s^c, a_s^w, a_s^c\}$ for $s \in \{t_1, \dots, T\}$. The parameters of the model are $\{\alpha, \delta, \sigma, \gamma_{ik}, d_{jik}, z_{ik}, \theta_{ik}, N, t_1, t_2, T, \mu, T^{soc}, \lambda, \beta, \nu_{med}, \nu_{slope}, \kappa, o, \underline{a}\}$ for countries $i, j \in \mathcal{I}$ and sectors $k \in \mathcal{K}$. The full description of the quantitative model and the definition of the steady state equilibrium can be found in Appendix B.3.

4 Quantitative Application

In this section I set up a quantitative application of the model to study external imbalances between Germany and the rest of Europe. I begin by motivating the exercise, proceed to describe calibration of the model, and finally outline counterfactuals to be performed.

4.1 European Imbalances Revisited

A number of explanations have been suggested as drivers behind the European imbalances. Many of these, including the low initial levels of capital stock, housing bubbles and implicit guarantees of sovereign debt can be classified as ‘pull factor’ theories, focusing on what attracted capital inflows into the net borrower economies (Blanchard and

¹⁶I retain the assumption of no fixed costs of entry, as non-zero costs of entry have no quantitatively meaningful effect on aggregate profits. In particular, introducing fixed costs of entry which halve the number of operating firms changes the aggregate profits by at most 0.02 percentage points, or 0.0002. This is due to the fat-tailed firm productivity distribution, where smallest firms that would exit in presence of entry contribute negligibly to the aggregate.

Giavazzi, 2002; Giavazzi and Spaventa, 2011; de Ferra, 2021). At the same time, as Figure 1 makes clear, persistent net lenders comprised the flip side of the European imbalances, making plenty finance available to the borrower states.¹⁷ In this section I ask: how much of the European imbalances can be attributed to a singular ‘push factor’ – the excess net asset demand due to high aggregate profit shares among the lender economies?

I further restrict my attention to one net lender economy: Germany. The choice is not coincidental. First, as Figure 1 shows, Germany is a good representation of the net foreign asset dynamics of the net lender group. Moreover, due to its sheer size,¹⁸ much of the European lending can be attributed to Germany directly: in 2019, its net foreign assets comprised over a half of all net foreign assets held by the net lender economies. In short, European imbalances are, to a large extent, German imbalances. Second, the German non-corporate sector stands out among its European peers both in terms of its aggregate profit share (ranking second in my sample) and the prevalence of ‘superstar’ firms (ranking first in terms of the thickness of the tails of its firm size distribution). Clearly, the behavior of German firms warrants attention. Third, Peter (2021) shows that German firms are also ‘closely held’: over 70% of firms in Germany are in private ownership, with a majority of these owned by one person. Likewise, publicly traded firms are dominated by insiders, with the top three shareholder equity shares adding to around 45%. Finally, there is a strong home bias: 88% of German portfolio investment is held by German investors. In short, German profits largely accrue to German entrepreneurs. Bringing these three observations together, then, in the rest of this paper I ask: what share of the net foreign asset position accumulated by Germany can be attributed to the profits mechanism?

¹⁷Much of this lending remained in Europe: between 2014 and 2019, over a half of the external assets held by net lender economies had a counter-party in Europe.

¹⁸Germany’s share of GDP among the net lenders is around 70%

4.2 Calibration

The model with free capital flows can be calibrated in blocks. The first block is the goods markets, where the competing firms determine the country-level income, aggregate profit shares, sectoral sales and trade flows. Since the interest rate is equalized across countries and since all firms produce with the same production function, the interest rate drops out from the firm sales share equation and is thus irrelevant for the equilibrium in the goods market. This property allows me to parameterize the firm productivity distributions and trade costs independently of the household side of the model. Once the production block is calibrated, I calibrate the parameters of the household side of the model.

Data and Methodology. To calibrate the production block, I need data on sectoral employment, bilateral trade flows, and tail parameters of firm sales distributions. The first two I obtain from the World Input-Output Database (WIOD). To calibrate tail parameters, I rely on the Orbis firm-level dataset. To calibrate the household block of the model I rely on data from the OECD, Penn World Tables, World Bank and the Household Finance and Consumption Survey (HFCS) compiled by the ECB. Since I calibrate the model to the financial liberalization steady state, I use the latest year before 2020 in each dataset.

I model the rest of Europe (RoE) as an aggregate of Belgium, Finland, France, Italy, Portugal, Spain and Sweden – the non-German economies available in my Orbis sample.

I calibrate the production and household blocks using simulated method of moments (SMM). Specifically, for a given parameter vector Θ , I simulate the model, compute a list of country-sector moments $M(\Theta)$, and compare these with the corresponding moments in the data \tilde{M} . I search for the parameter vector that minimizes the distance between the model and the empirical moments, according to the loss function $L(\Theta) = (M(\Theta) - \tilde{M})' (M(\Theta) - \tilde{M})$. See Appendix B.5 for details.

Production Block. I begin by constructing the production, consumption, and trade flows between Germany and the RoE. The model abstracts from the input-output structure of

production, and from all other economies. To model this in the data, I use the absorption variable in WIOD (sum of final and intermediate expenditure shares) which varies by sector, consumer country and producer country. I discard all absorption from and by economies outside of my sample, and construct RoE absorption as the sum of my seven RoE economies. In Appendix B.4 I show how to use absorption to construct Germany and RoE GDP, sectoral expenditure shares, and trade flows.

External Calibration: I set the number of sectors to 42, the first 41 of these are the non-financial corporate sectors covered by WIOD.¹⁹ The remaining sector is an aggregate of all other sectors in the economy: utilities, finance, real estate, and government funded sectors of the economy. I set the number of firms per two-digit sector to $N = 1000$ for the RoE. I set the number of firms in Germany to 500.²⁰ Since the outer nest of the utility function is Cobb-Douglas, I set γ_{ik} to equal the sectoral expenditure shares in my data. I set α – the share of capital in production – to 0.23, to match the average labor share of 66% and profit share of 14.5% in the sample.²¹ I take L directly from WIOD, aggregating across European economies for the RoE. This leaves $\sigma, d_{jik}, z_{ik}, \theta_{ik}$ to estimate.

Internal Calibration: $\sigma, d_{jik}, z_{ik}, \theta_{ik}$ can be estimated targeting four objects: trade flows between Germany and the RoE, country-sector output per worker, the logarithms of Herfindahl–Hirschman Indices of firm size distributions in Orbis,^{22,23} and the average aggregate profit share in my sample. Intuitively, trade costs are pinned down by the observed trade flows, scale parameters are pinned down by output per worker, tail pa-

¹⁹ A01, A02, A03, B, C10-C12, C13-C15, C16, C17, C18, C20, C21, C22, C23, C24, C25, C26, C27, C28, C29, C30, C31-C32, C33, F, G45, G46, G47, H49, H50, H51, H52, H53, I, J58, J59-J60, J61, J62-J63, M69-M70, M71, M72, M73, M74-M75.

²⁰ German population constitutes 50% of RoE aggregate.

²¹ $rK/(wL + rK) = (Y - wL - \Pi)/wL = (1 - 0.66 - 0.145)/(1 - 0.145) \approx 0.23$.

²² I use HHI estimated in my Orbis data as a moment describing firm size distribution and not the tail parameters estimated using SBS employment by size bins data as the former can be computed without the reference to particular firm size thresholds, which are difficult to map to in the context of the model. If the underlying distribution is Pareto, HHI is a sufficient statistic for the tail parameter θ : $HHI = \zeta_N(2\theta)/\zeta_N(\theta)^2$, where ζ_N is the truncated Riemann Zeta function (Naldi and Flamini, 2014). I use average HHI across years for each sector for Germany, and average HHI across years and countries for the RoE.

²³ I assume all firms in the residual sector of the economy are equally productive, which implies their market shares are equal and therefore the sector features minimal market power.

parameters are pinned down by the observed HHI, and finally, elasticity of substitution reconciles the observed HHI and the average aggregate profit share in the data.

Household Block. In calibrating the household side, I choose a common set of parameters for Germany and the RoE so that the simulated external imbalances are driven solely by the heterogeneity on the firm side. Thus, I target the average moments in my sample.

External Calibration: I set the age of entry to the labor force, t_1 , to 27 years – the age at which half the age-cohort is in full-time employment. I set the age of retirement, t_2 , to 64 and the age of death, T , to 80 – representing the averages in my sample. I pick T^{soc} to match the average net replacement ratio of 0.70.²⁴ This gives rise to pension expenditure of 15% of GDP, compared to 13% in the data. I set the share of capitalists, μ , to 7.6%, the discount factor to 0.98, both following Peter (2021), and the income elasticity of consumption at the median age, ν_{med} , to 2.5 as in Straub (2018). I set δ , the rate of capital depreciation, to 4%, targeting that in the PWT. Finally, I set λ , the parameter governing the pledgeability of profit streams, to 0.08, targeting the average stock market capitalization of 56% in my sample.²⁵

Internal Calibration: I calibrate the remaining four parameters – $\kappa, \underline{a}, \theta, \nu_{slope}$ – targeting (i) bequests as a share of GDP of 6.85% (following Alvarado et al. (2017), who estimate values of 7.2% for France and 6.5% for Germany), (ii) the share of assets held by the top 7.5% of households, which is 43.2% in my sample,²⁶ (iii) the aggregate assets-to-GDP ratio that, together with the depreciation rate of 4% results in an interest rate of 2% – the average corporate interest rate in 2019 in my sample, and (iv) the ratio of bequests between capitalists and workers of 7.5.²⁷ See Table 2 for the list of all parameters.

Model Fit. On both the production and household side, I have a matching number of

²⁴The ratio of pension entitlement to pre-retirement earnings net of social security contributions.

²⁵The stock market capitalization in Germany is 54% in 2019, so setting the value between Germany and the RoE equal is quantitatively inconsequential.

²⁶HFCS only records the net wealth held by top-10% and top-5%. 43.2% is the average of these two statistics across all European economies in the sample.

²⁷I obtain the ratio from Hurd et al. (2001), who report the distribution of bequests left by single decedents. Authors find the 95th and 50th percentiles at 250000\$ and 33300\$ respectively, giving a ratio of 7.5.

Table 2: Baseline calibration

Parameter	Description	Value	Target/Source
Production			
α	Capital share	0.23	WIOD
δ	Depreciation rate	0.04	PWT
σ	Within-sector elasticity of substitution	9.4	Aggregate profit share, OECD
γ_{ik}	Cobb-Douglas shares in final production	Vector	Sectoral absorption, WIOD
d_{jik}	Bilateral trade costs	Matrix	Bilateral trade flows, WIOD
Firm distribution			
z_{ik}	Productivity Pareto scale parameter	Matrix	Real output per worker, WIOD
θ_{ik}	Productivity Pareto tail parameter	Matrix	HHI_{ik} , Orbis
N	Number of firms per sector in the RoE	1000	
Population			
t_1	Age of entry into the labor force	27	OECD
t_2	Age of retirement	64	OECD
T	Age of death	80	OECD
μ	Share of capitalists	0.076	Peter (2021)
T^{soc}	Net replacement ratio	0.70	OECD
λ	Pledgeability of future profits	0.08	Market capitalization, World Bank
Preferences			
β	Discount factor	0.98	Peter (2021)
v_{med}	Elasticity of intertemporal substitution	2.5	Straub (2018)
v_{slope}	Ratio of elasticities v_{s+1}/v_s	0.99	Hurd et al. (2001)
k	Weight on bequest motive	39	Alvaredo et al. (2017)
o	Scale term in utility function	5.5% of GDP	$r = 0.02$, PWT
a	Intercept in bequest utility	0.025	Net wealth of top 7.5%, HFCS

moments and parameters, resulting in an exact fit.

Untargeted moments: There are two categories of untargeted moments in the calibration. First, while I target the average aggregate profit share, the individual countries' profit shares are not a targeted moment. The baseline calibration delivers a profit share of 16% for Germany and 13.7% for the RoE, compared to 17% and 13.4% in the data respectively. Thus, the model matches country-level aggregate profit shares surprisingly well. Note further that in the stylized model the external positions are proportional to the profit share gap. Thus, since my calibration errs on the side of lower variability, the simulation gives rise to a conservative estimate of the external imbalances due to the profit mechanism.

The second group of untargeted moments concerns the household side of the model.

First, while I use the asset side inequality to calibrate the strength of the non-homotheticity in the utility function, the income inequality is not targeted. Nevertheless, the ratio of income of the two groups falls close to the the ratio of 95th and 50th percentiles of incomes in the data – at 3.4 and 3.3 respectively. Second, the key statistic in [Straub \(2018\)](#) – the propensity to consume out of permanent income (obtained by regressing consumption on permanent income, controlling for age) – was not targeted. Despite this, my model gives rise to a value of 0.76, remarkably close to the 0.7 reported in [Straub \(2018\)](#). In short, while the households remain stylized, the quantitative model is successful at capturing, in very broad strokes, the pattern of household heterogeneity in the data.

4.3 Counterfactual Exercises

To study European imbalances through the prism of the model, I make several assumptions. First, I assume that financial autarky is long over by the beginning of the period studied. This decision reflects the fact that many European economies recorded non-zero net foreign asset positions in the 1980s – the earliest years for which the data is available. Next, I assume that in 2019 Europe had reached the financial liberalization steady state. While an abstraction, this assumption rests on two observations. First, the average long-run interest rate spreads against Germany in my rest of Europe sample were below 1% from 2015 onwards. Second, while total German net foreign assets were still on an upward trajectory in 2019, the bilateral holdings of portfolio assets and foreign direct investment between Germany and the RoE aggregate have flat-lined between 2013 and 2019.²⁸ Inasmuch as 2019 constitutes a transition towards a steady state with higher net foreign asset holdings in Germany, the contribution of the profit mechanism computed in Section 5 needs to be viewed as an upper bound.

With this mapping in mind, I conduct three exercises. First, I simulate the model un-

²⁸Net foreign asset position can be decomposed into the net holdings of portfolio assets, FDI, debt, financial derivatives and foreign exchange reserves. IMF maintains a database of bilateral positions of portfolio and FDI and thus enables partial reconstruction of the bilateral net asset holdings.

der financial liberalization calibrated to the 2019 steady state and measure the net foreign asset position of Germany vis-à-vis the RoE aggregate. This exercise addresses a stark question: what was the contribution of the profit mechanism in driving European imbalances in a counterfactual scenario where all alternative drivers of external imbalances have been switched off? Second, I ask: what was the relative contribution of the asset supply and asset demand channels in driving the German imbalances? This question can be addressed in the same simulation by comparing the difference in asset supply and asset demand between Germany and the RoE aggregate. Third, I ask: what are the dimensions of heterogeneity responsible for the imbalances generated in the model? To answer this question I re-calibrate the model making Germany and the RoE differ by one set of parameters at a time and compare the resulting external positions against the baseline.

Finally, I conclude the paper by asking how the United States fits the story by constructing a stylized model of the imbalances between the United States and rest of world. The exercise highlights the interaction between the relatively poorly developed financial markets in Europe and the profits mechanism in driving the European imbalances.

5 Results

Table 3 summarizes the results of the main exercise: the simulation of the financial liberalization steady state between Germany and the RoE as calibrated in Section 4.2. The model is successful in matching the European imbalances qualitatively, with positive net foreign asset position in Germany and net foreign debt in the RoE. Quantitatively, the mechanism generates net foreign assets of 14% of GDP in Germany, and -6% in the RoE. This result corresponds to 24% – roughly a quarter – of the net foreign assets held by Germany. Thus, I argue that the profit mechanism is an important driver of European imbalances.

Columns 5 to 8 break down the net foreign asset positions of the two regions into the demand for assets, supply of assets, and the latter further by type of asset: capital

Table 3: Simulation results: Profits and External Imbalances, Germany vs the RoE

	Profit Share		Net Foreign Assets		Asset Demand	Asset Supply	Physical Capital	Financial Assets
	Model	Data	Model	Data	Model			
Germany	0.16	0.17	0.14	0.59	3.59	3.45	2.96	0.49
RoE	0.14	0.14	-0.06	-0.22	3.40	3.46	3.04	0.42

Note: All variables in percent of GDP. The first two columns present the aggregate profit shares in the model and in the data as computed in Section 2.4, the third and fourth columns present the net foreign assets in the model and in the data in 2019. Columns five to eight present the demand for assets, supply of assets, and the breakdown of the latter into physical capital and financial assets in the simulation.

and the financial wealth generated by the contractible part of the future profit streams, all as a share of GDP. The results are instructive: the reason behind Germany’s positive external position in the model is its higher demand for assets. At the same time, its asset supply is comparable to that in the RoE. This result arises due to the opposing effects of profits on the firm’s demand for capital and the financial wealth they generate. While Germany features a slightly lower capital-to-GDP ratio than the RoE, it also generates a slightly larger supply of financial assets. Quantitatively, the two effects offset almost exactly. Thus, the profit mechanism operates mainly via the asset demand channel.

Finally, I explore the contribution of different types of heterogeneity on the production side – firm distribution scale and tail parameters \bar{z} and θ , trade costs d , final consumption expenditure shares γ and the population size L – to the baseline result. To do so, I recalibrate the model, setting all parameter values to a simple average between Germany and the RoE. This specification gives rise to balanced external positions by construction. I then return parameter values to their baseline calibration values one at a time, and compute the parameter contribution as the ratio of the German net foreign asset position in the new simulation to that in the baseline. The results are displayed in Table 4. I find that the imbalances in the model are mainly driven by the difference in the tail parameters of the firm productivity distributions. Note that this is in line with the predictions of the stylized model: external imbalances depend on the aggregate profit share gap, which in

turn is pinned down by the relative prevalence of firms with extreme productivity draws, that is, the thickness of the tails. Thus, I confirm this result in a quantitative setting.

Table 4: Contribution of Different Sources of Heterogeneity

	Variable	Contribution
Productivity scale	\bar{z}	1.1
Productivity tail	θ	70.7
Trade costs	d	12.7
Expenditure shares	γ	17
Population	L	4

Note: Values in the ‘Contribution’ column are the ratio of the German net foreign asset position as a share of GDP in the restricted simulation to that in the baseline, expressed in percentage points. In all simulations, I set each of $\bar{z}, \theta, d, \gamma, L$ to a simple average between Germany and the RoE with the exception of the parameter indicated in the row.

Global Imbalances? I conclude by bringing the framework developed in this paper to the net borrowing of the United States, and the corresponding net lending by the rest of the world. The reason for this is two-fold. First, net borrowing by the United States is extraordinary in its scale and persistence, spurring a literature on global imbalances. Second, United States also is home to many of the world’s largest firms. What is the role of profits generated by these companies for the external position of the United States?

I explore this question in a stylized exercise. I focus on two economies – United States and the rest of the world (RoW), calibrating the production side as outlined in Appendices B.4 and B.5. I take the average sectoral HHI in my Orbis sample to be representative of the rest of the world, and assume that the US exhibits a higher degree of concentration: 125%, 150% or 175% of the sectoral-level HHI in the RoW.²⁹ I retain the parameterization of the household side of the model as in the baseline, with one exception: I now calibrate λ , the parameter governing the share of future profits that are pledgeable as a financial asset separately for the US and RoW. Specifically, I target the level of market capitalization of 105% of GDP for the RoW, and 160% of GDP for the United States (2019 values).

²⁹Tight calibration to the economies of the U.S. and RoW is beyond the scope of this paper, the aim of the exercise is to illustrate the interaction of the profits mechanism with the depth of financial markets.

The results are presented in Table 5. For each of the three levels of concentration the United States holds net foreign debt. Note further that the level of debt is comparable to that in the data – 30% in the median calibration (150% higher HHI) compared to 55% in the data. Thus, higher profit share does not make the United States a net lender. Why not? A breakdown of the net foreign asset position under the median scenario into the asset demand and asset supply clarifies this result. While the United States holds more assets, its financial system is more capable of generating stores of value out of the future profit streams. Combined with a higher profit share, this means that the United States generates more financial assets than the RoW. As a result, the United States holds net foreign debt.

Table 5: Simulation results: Profits and External Imbalances, United States vs RoW

	Net Foreign Assets			Asset	Asset	Physical	Financial
	Calib. #1	Calib. #2	Calib. #3	Demand	Supply	Capital	Assets
United States	-0.34	-0.30	-0.24	3.74	4.04	2.82	1.22
RoW	0.06	0.05	0.04	3.71	3.65	2.84	0.82

Note: All variables in percent of GDP. The first three columns present the net foreign assets in calibrations where the United States has HHI of 125%, 150% and 175% of that in RoW. Columns four to seven present the demand for assets, supply of assets, and the breakdown of the latter into physical capital and financial assets in the simulation where HHI indices in the United States are 150% of that in RoW.

6 Conclusion

I document a novel stylized fact: that European lender economies feature higher profit shares and a higher prevalence of ‘superstar’ firms. I suggest a novel theory that generates such a relationship endogenously and use a quantitative application to argue that the profit mechanism has contributed to the development of the European imbalances.

There is renewed recognition that ‘fickle’ capital flows can be damaging to recipient economies (Caballero and Simsek, 2020). In the world increasingly characterized by ‘superstar’ firm dynamics (Autor et al., 2020), it is thus imperative to understand how the unequal distribution of such firms contribute to the buildup of external imbalances.

References

- ALVAREDO, F., B. GARBINTI, AND T. PIKETTY (2017): "On the share of inheritance in aggregate wealth: Europe and the USA, 1900–2010," *Economica*, 84, 239–260.
- ATKESON, A. AND A. BURSTEIN (2008): "Pricing-to-market, trade costs, and international relative prices," *American Economic Review*, 98, 1998–2031.
- ATKESON, A., J. HEATHCOTE, AND F. PERRI (2022): "The end of privilege: A reexamination of the net foreign asset position of the united states," Tech. rep., National Bureau of Economic Research.
- AUTOR, D., D. DORN, L. F. KATZ, C. PATTERSON, AND J. VAN REENEN (2020): "The fall of the labor share and the rise of superstar firms," *The Quarterly Journal of Economics*, 135, 645–709.
- BAJGAR, M., G. BERLINGIERI, S. CALLIGARIS, C. CRISCUOLO, AND J. TIMMIS (2020): "Coverage and representativeness of Orbis data," .
- BARKAI, S. (2020): "Declining labor and capital shares," *The Journal of Finance*, 75, 2421–2463.
- BERNARD, A. B., J. EATON, J. B. JENSEN, AND S. KORTUM (2003): "Plants and productivity in international trade," *American economic review*, 93, 1268–1290.
- BLANCHARD, O. AND F. GIAVAZZI (2002): "Current account deficits in the euro area: the end of the Feldstein-Horioka puzzle?" *Brookings papers on economic activity*, 2002, 147–209.
- BURSTEIN, A., V. M. CARVALHO, AND B. GRASSI (2020): "Bottom-up markup fluctuations," Tech. rep., NBER.
- CABALLERO, R. J., E. FARHI, AND P.-O. GOURINCHAS (2008): "An equilibrium model of" global imbalances" and low interest rates," *American economic review*, 98, 358–93.
- CABALLERO, R. J. AND A. SIMSEK (2020): "A model of fickle capital flows and retrenchment," *Journal of Political Economy*, 128, 2288–2328.
- CARROLL, C. D. (2000): "Why do the rich save so much?" in *Does Atlas Shrug? The Economic Consequences of Taxing the Rich*, ed. by J. B. Slemrod, Cambridge: Harvard University Press, 466–484.

- CHEN, Z. (2022): "Economic Growth and the Rise of Large Firms," Working paper.
- DE FERRA, S. (2021): "External Imbalances, Gross Capital Flows, and Sovereign Debt Crises," *Journal of the European Economic Association*, 19, 347–402.
- DE LOECKER, J., J. EECKHOUT, AND G. UNGER (2020): "The rise of market power and the macroeconomic implications," *The Quarterly Journal of Economics*, 135, 561–644.
- DE NARDI, M. (2004): "Wealth inequality and intergenerational links," *The Review of Economic Studies*, 71, 743–768.
- DE RIDDER, M., B. GRASSI, G. MORZENTI, ET AL. (2022): "The Hitchhiker's Guide to Markup Estimation," .
- DYNAN, K. E., J. SKINNER, AND S. P. ZELDES (2004): "Do the rich save more?" *Journal of political economy*, 112, 397–444.
- EDMOND, C., V. MIDRIGAN, AND D. Y. XU (2023): "How costly are markups?" *Journal of Political Economy*, 131, 1619–1675.
- FAGERENG, A., M. B. HOLM, B. MOLL, AND G. NATVIK (2019): "Saving behavior across the wealth distribution: The importance of capital gains," Tech. rep., NBER.
- FERRERO, A. (2010): "A structural decomposition of the US trade balance: Productivity, demographics and fiscal policy," *Journal of Monetary Economics*, 57, 478–490.
- FISHER, I. (1930): *Theory of interest: as determined by impatience to spend income and opportunity to invest it*, Augustusm Kelly Publishers, Clifton.
- GAUBERT, C. AND O. ITSKHOKI (2021): "Granular comparative advantage," *Journal of Political Economy*, 129, 871–939.
- GIAVAZZI, F. AND L. SPAVENTA (2011): "11 Why the current account may matter in a monetary union: lessons from the financial crisis in the euro area," *The Euro area and the financial crisis*.
- GOPINATH, G., Ş. KALEMLI-ÖZCAN, L. KARABARBOUNIS, AND C. VILLEGAS-SANCHEZ (2017): "Capital allocation and productivity in South Europe," *The Quarterly Journal of Economics*, 132, 1915–1967.
- GOURINCHAS, P.-O. AND H. REY (2014): "External adjustment, global imbalances, valuation effects," in *Handbook of international economics*, Elsevier, vol. 4, 585–645.

- HURD, M. D., J. P. SMITH, ET AL. (2001): "Anticipated and actual bequests," *Themes in the Economics of Aging*, 357–389.
- KALEMLI-OZCAN, S., E. PAPAIOANNOU, AND J.-L. PEYDRÓ (2010): "What lies beneath the euro's effect on financial integration? Currency risk, legal harmonization, or trade?" *Journal of International Economics*, 81, 75–88.
- KALEMLI-OZCAN, S., B. SORENSEN, C. VILLEGAS-SANCHEZ, V. VOLOSOVYCH, AND S. YESILTAS (2015): "How to construct nationally representative firm level data from the Orbis global database: New facts and aggregate implications," Tech. rep., NBER.
- LANE, P. R. AND G. M. MILESI-FERRETTI (2018): "The external wealth of nations revisited: international financial integration in the aftermath of the global financial crisis," *IMF Economic Review*, 66, 189–222.
- LIU, E., A. MIAN, AND A. SUFI (2022): "Low interest rates, market power, and productivity growth," *Econometrica*, 90, 193–221.
- MENDOZA, E. G., V. QUADRINI, AND J.-V. RIOS-RULL (2009): "Financial integration, financial development, and global imbalances," *Journal of Political economy*, 117, 371–416.
- MIAN, A., L. STRAUB, AND A. SUFI (2021): "Indebted demand," *The Quarterly Journal of Economics*, 136, 2243–2307.
- NALDI, M. AND M. FLAMINI (2014): "Interval estimation of the Herfindahl-Hirschman index under incomplete market information," in *2014 UKSim-AMSS 16th International Conference on Computer Modelling and Simulation*, IEEE, 318–323.
- PETER, A. (2021): "Equity Frictions and Firm Ownership," Tech. rep., Working Paper.
- REIS, R. (2013): "The Portuguese slump and crash and the euro crisis," Tech. rep., NBER.
- STRAUB, L. (2018): "Consumption, savings, and the distribution of permanent income," *Unpublished manuscript, Harvard University*.

A Proofs

A.1 Proof of Proposition 1

Begin with the derivative of profit share with respect to a shock in firm i productivity:

$$\frac{d\pi}{dz_i} = \frac{\sigma - 1}{\sigma} \frac{\sum_i s_i^2}{\sum_i s_i} \sum_i (2h_i - d_i) \frac{ds_i}{dz_i} \frac{1}{s_i},$$

where $h_i = s_i^2 / \sum_j s_j^2$ and $d_i = s_i / \sum_j s_j$. Sales shares respond to the change directly, but also to the changes in the relative factor costs $\omega = \left(\frac{w}{w^*}\right)^{(1-\alpha)} \left(\frac{r}{r^*}\right)^\alpha$:

$$\begin{aligned} \frac{ds_i}{dz_i} \frac{1}{s_i} &= \left(\frac{1}{z_i} - \frac{s_i v(s_i)}{v^G} \frac{1}{z_i} - \frac{v^*}{v^G} \frac{d\omega}{dz_i} \frac{1}{\omega} \right) v(s_i), \\ \frac{ds_j}{dz_i} \frac{1}{s_j} &= - \left(\frac{s_i v(s_i)}{v^G} \frac{1}{z_i} + \frac{v^*}{v^G} \frac{d\omega}{dz_i} \frac{1}{\omega} \right) v(s_j), \\ \frac{ds_j^*}{dz_i} \frac{1}{s_j^*} &= - \left(\frac{s_i v(s_i)}{v^G} \frac{1}{z_i} - \frac{v}{v^G} \frac{d\omega}{dz_i} \frac{1}{\omega} \right) v(s_j^*), \text{ where} \end{aligned}$$

$$v(s_i) = \left((\sigma - 1)^{-1} + \frac{s_i}{1 - s_i} \right)^{-1}, \quad v^* = \sum_i s_i v(s_i), \quad v^* = \sum_i s_i^* v(s_i^*), \quad v^G = v + v^*.$$

Plugging in,

$$\frac{d\pi}{dz_i} = \frac{\sigma - 1}{\sigma} \frac{\sum_i s_i^2}{\sum_i s_i} \left[(2h_i - d_i) v(s_i) \frac{1}{z_i} - \sum_j (2h_j - d_j) v(s_j) \left(\frac{s_i v(s_i)}{v^G} \frac{1}{z_i} + \frac{v^*}{v^G} \frac{d\omega}{dz_i} \frac{1}{\omega} \right) \right].$$

Suppose the economies are in financial autarky. In this case, the relative factor costs are:

$$\omega = \frac{(1 - \pi)Y}{(1 - \pi^*)Y^*}, \quad \text{and} \quad \frac{d\omega}{dz_i} = \omega \left(-\frac{d\pi}{dz_i} \frac{1}{1 - \pi} + \frac{d\pi^*}{dz_i} \frac{1}{1 - \pi^*} + \frac{dY/Y^*}{dz_i} \frac{1}{Y/Y^*} \right),$$

where

$$\frac{d\pi}{dz_i} = \frac{\sigma - 1}{\sigma} \frac{\sum_i s_i^2}{\sum_i s_i} \sum_i (2h_i - d_i) \frac{ds_i}{dz_i} \frac{1}{s_i},$$

$$\begin{aligned}\frac{d\pi^*}{dz_i} &= \frac{\sigma - 1}{\sigma} \frac{\sum_i s_i^2}{\sum_i s_i^*} \sum_i (2h_i^* - d_i^*) \frac{ds_i^*}{dz_i} \frac{1}{s_i^*}, \\ \frac{dY/Y^*}{dz_i} \frac{1}{Y/Y^*} &= \sum_i d_i \frac{ds_i}{dz_i} \frac{1}{s_i} - \sum_i d_i^* \frac{ds_i^*}{dz_i} \frac{1}{s_i^*}.\end{aligned}$$

Plugging in and combining with the sales share derivative equations,

$$\begin{aligned}\frac{d\omega}{dz_i} \frac{1}{\omega} &= b_i \frac{1}{z_i} - b \left(\frac{s_i v(s_i)}{v^G} \frac{1}{z_i} + \frac{v^*}{v^G} \frac{d\omega}{dz_i} \frac{1}{\omega} \right) + b^* \left(\frac{s_i v(s_i)}{v^G} \frac{1}{z_i} - \frac{v}{v^G} \frac{d\omega}{dz_i} \frac{1}{\omega} \right) = \\ &= \frac{b_i v^G - (b - b^*) s_i v(s_i)}{v^G + b v^* + b^* v} \frac{1}{z_i}, \quad \text{where } b_i = d_i v(s_i) - \frac{\sigma - 1}{\sigma} \frac{1}{1 - \pi} \frac{\sum_i s_i^2}{\sum_i s_i} (2h_i - d_i) v(s_i), \\ b &= \sum_i b_i, \quad \text{and } b^* = \sum_i \left(d_i^* v(s_i^*) - \frac{\sigma - 1}{\sigma} \frac{1}{1 - \pi^*} \frac{\sum_i s_i^2}{\sum_i s_i^*} (2h_i^* - d_i^*) v(s_i^*) \right).\end{aligned}$$

Plugging back into the derivative of profit share,

$$\frac{d\pi}{dz_i} = \frac{\sigma - 1}{\sigma} \frac{\sum_i s_i^2}{\sum_i s_i} \left[(2h_i - d_i) v(s_i) - \sum_j (2h_j - d_j) v(s_j) \left(\frac{s_i v(s_i) + b_i v^* + b^* s_i v(s_i)}{v^G + b v^* + b^* v} \right) \right] \frac{1}{z_i}.$$

Note that $\frac{\sigma - 1}{\sigma} \frac{\sum_i s_i^2}{\sum_i s_i} (2h_i - d_i) v(s_i) = (1 - \pi) (d_i v(s_i) - b_i)$. Plugging in,

$$\begin{aligned}\frac{d\pi}{dz_i} &= \frac{1 - \pi}{v^G + b v^* + b^* v} \left[\left(\sum_i d_i v(s_i) - b \right) (s_i v(s_i) (1 + b^*) + v^* b_i) (d_i v(s_i) - b_i) (v^G + b v^* + b^* v) \right] \\ &= \frac{(1 - \pi)}{v^G + b v^* + b^* v} \left[\frac{v^*}{Y} (s_i v(s_i) - b_i Y) + \left(1 + b^* + \frac{v^*}{Y} \right) (s_i v(s_i) b - b_i v) \right] \\ &= -\frac{(1 - \pi)}{v^G + b v^* + b^* v} \left[\frac{v^* s_i v(s_i)}{Y(1 - s)} (2s_i - s) + \left(1 + b^* + \frac{v^*}{Y} \right) \frac{2s_i v(s_i)}{Y(1 - s)} \left(\sum_j s_j v(s_j) (s_i - s_j) \right) \right].\end{aligned}$$

$1 + b > 0, 1 + b^* > 0$, so the denominator in the first term is positive.

For $s_i = s_1, 2s_1 \geq s$ and $s_1 \geq s_j$ for all $j \in \mathcal{N}$, so the square bracket is positive and $\frac{d\pi}{dz_1} > 0$.

For $s_i = s_N, s_N \leq s_j$ for all $j \in \mathcal{N}$, so the last term is non-positive for the least productive firm. A sufficient condition for the whole expression in the square brackets to be non-positive is that $2s_N \leq s$, in which case $\frac{d\pi}{dz_N} \leq 0$.

A.2 Proof of Proposition 2

First note that capital demand decreases in aggregate profit share:

$$\frac{K}{Y} = \frac{\alpha}{r_a}(1 - \pi), \quad \frac{d\frac{K}{Y}}{d\pi} = -\frac{\alpha}{r_a} < 0.$$

Aggregate asset demand, on the other hand, increases in aggregate profit share:

$$\begin{aligned} \frac{A}{Y} &= \zeta_c(\mu(1 - \alpha)(1 - \pi) + \pi) + \zeta_w(1 - \mu)(1 - \alpha)(1 - \pi), \\ \frac{d\frac{A}{Y}}{d\pi} &= (\zeta_c - \zeta_w)(1 - \mu(1 - \alpha)) + \zeta_w\alpha > 0. \end{aligned}$$

Finally, autratic interest rate decreases in the aggregate profit share:

$$\begin{aligned} r_a &= \frac{\alpha(1 - \pi)}{\zeta_c(\mu(1 - \alpha)(1 - \pi) + \pi) + \zeta_w(1 - \mu)(1 - \alpha)(1 - \pi)}, \\ \frac{dr_a}{d\pi} &= -\alpha \left(\frac{A}{Y}\right)^{-2} \left(\frac{A}{Y} + (1 - \pi)\frac{d\frac{A}{Y}}{d\pi}\right) < 0. \end{aligned}$$

A.3 Proof of Proposition 3

The global interest rate clears the global asset market:

$$\begin{aligned} A + A^* &= \left(\zeta_c(\mu(1 - \alpha)(1 - \pi^G) + \pi^G) + \zeta_w(1 - \mu)(1 - \alpha)(1 - \pi^G)\right) = \\ &\eta \frac{\alpha}{r_a}(1 - \pi) + (1 - \eta) \frac{\alpha}{r_G}(1 - \pi^*) = \frac{\alpha}{r_G}(1 - \pi^G) = K + K^*, \quad \rightarrow \\ r^G &= \frac{\alpha(1 - \pi^G)}{\zeta_c(\mu(1 - \alpha)(1 - \pi^G) + \pi^G) + \zeta_w(1 - \mu)(1 - \alpha)(1 - \pi^G)}. \end{aligned}$$

Net foreign assets are the difference between asset demand and capital as a share of GDP:

$$\begin{aligned} \frac{A - K}{Y} &= \zeta_c(\mu(1 - \alpha)(1 - \pi) + \pi) + \zeta_w(1 - \mu)(1 - \alpha)(1 - \pi) - \\ &\frac{1 - \pi}{1 - \pi^G} \left(\zeta_c(\mu(1 - \alpha)(1 - \pi^G) + \pi^G) + \zeta_w(1 - \mu)(1 - \alpha)(1 - \pi^G)\right) = \zeta_c(1 - \eta) \frac{(\pi - \pi^*)}{1 - \pi^G}. \end{aligned}$$

B Online Appendix

B.1 Description of the Dataset

Tail. I construct the *tail* variable using EMPN/Total employment (persons employed) by size bin from Structural Business Statistics ISIC Rev. 4 dataset by OECD. I remove all country-size-sector observations where any of the years is flagged as a structural break. I stop analysis in 2017 since in 2018 the share of observations with structural breaks jumps seven-fold (from around 5% per year to around 35%). I construct *tail* as

$$tail = \log \frac{\tilde{F}(T_L)}{\tilde{F}(T_S)} / \log \frac{T_L}{T_S},$$

where I set T_L to 250+ employees, T_S to 10+ or 50+ employees, and $\tilde{F}(x)$ sums over all firms in a given sector, country and year with employees above the specified threshold. I compute *tail* for 39 two-digit ISIC non-financial corporate sectors³⁰ and 28 economies³¹. I compute \overline{tail} as the simple average of *tail* for all country-year pairs with more than a quarter of industries featuring non-NaN *tail* estimates. Dataset covers years 2005-2017.

Aggregate profit. I construct my main measure of aggregate profit as follows:

$$PR_{it} = GOS_{it} - (r_{it} + \delta_{it})K_{it},$$

where GOS_{it} is the gross operating surplus (B2G) of the non-financial corporate sector from OECD sectoral Annual Non-financial Accounts database (ANFA), r is the *bank interest rate – loans to corporations (outstanding amounts)* variable from MFI Interest Rate Statis-

³⁰A01, A02, A03, B05, B06, B07, B08, B09, C10-C12, C13-C15, C16, C17, C18, C20, C21, C22, C23, C24, C25, C26, C27, C28, C29, C30, C31-C32, C33, F, G45, G46, G47, H49, H50, H51, H52, H53, I, J58, J61, M71, M72, M73, N. Here and elsewhere, I exclude utilities, government provided services, finance and real estate when constructing variables for the non-financial corporate sector.

³¹Austria, Belgium, Bulgaria, Costa Rica, Czech Republic, Germany, Denmark, Spain, Estonia, Finland, France, Greece, Hungary, Israel, Italy, Japan, Lithuania, Latvia, North Macedonia, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Sweden, Turkey. I exclude South Korea, whose average *tail* estimate lies over three standard deviations below the mean in my sample.

tics compiled by the ECB,³² δ is the *depreciation rate* variable from Penn World Tables, and K is the sum of the capital stocks of the non-financial corporate sectors in World Input Output Database 2016. Since WIOD coverage ends in 2014, for later years I construct K recursively, by adding the gross fixed capital formation variable from ANFA ($P5$) and subtracting the product of last period's capital stock and depreciation rates. Since the interest rate is from the ECB, this measure of profits is available only for European economies.

For my secondary measure of aggregate profit I use *entrepreneurial income* variable from ANFA, $B4G$. The entrepreneurial income is a close approximation to before tax profits in business accounting. Whenever $B4G$ is missing, I construct entrepreneurial income following the System of National Accounts 2008 definition:

$$B4G_{it} = B2G_{it} + D4(C)_{it} - D41(D)_{it} - D45(D)_{it},$$

where $B2G$ is the gross operating surplus, $D4(C)$ is the property income receivable, $D41(D)$ and $D45(D)$ are the interest and rent payable, respectively. Since this variable can be constructed from ANFA data alone, it is available for a wider sample of economies.³³ The correlation between the two measures of profit shares (profits as a % of GDP) is 0.87.

Other variables. I obtain *Net Foreign Assets (% GDP)* from External Wealth of Nations database (Lane and Milesi-Ferretti, 2018), *Rule of Law* index from Worldwide Governance Indicators by World Bank³⁴, *population growth* from World Bank database, *old-age to working-age ratio* from OECD's Pensions at a glance database.

³²Results are robust to using interest rates on new loans instead.

³³Austria, Belgium, Brazil, Switzerland, Colombia, Costa Rica, Czech Republic, Germany, Spain, Estonia, Finland, France, Greece, Hungary, Italy, Lithuania, Latvia, Mexico, Netherlands, Norway, Portugal, Romania, Slovenia, Sweden, Turkey, and the United States.

³⁴Rule of law captures perceptions of the extent to which agents have confidence in and abide by the rules of society, and in particular the quality of contract enforcement, property rights, the police, and the courts, as well as the likelihood of crime and violence.

B.2 Additional Empirical Results

Table B.1: European Economies: Alternative Variable Definitions

	Agg. Profit (EI) (% GDP)		Net Foreign Assets (% GDP)		
	(1)	(2)	(3)	(4)	(5)
\overline{tail}_{50}	0.399** (0.152)	5.992*** (1.097)	4.066*** (1.080)		
pr_{EI}				5.549* (2.217)	4.772* (2.304)
FE	Y	Y	Y	Y	Y
Clustering	C	C	C	C	C
Controls			✓		✓
Observations	94	96	96	265	265
R ²	0.329	0.496	0.715	0.342	0.398

Note: Here, I use 50 employees as the small firm cutoff for computing the *tail*, and entrepreneurial income as a measure of aggregate profit (EI). *p<0.1; **p<0.05; ***p<0.01.

Table B.2: All Countries

	Agg. Profit (EI) (% GDP)		Net Foreign Assets (% GDP)		
	(1)	(2)	(3)	(4)	(5)
\overline{tail}	0.137 (0.264)	3.843* (1.633)	3.266** (1.026)		
pr_{EI}				4.026* (1.823)	3.133+ (1.739)
FE	Y	Y	Y	Y	Y
Clustering	C	C	C	C	C
Controls			✓		✓
Observations	187	245	218	462	446
R ²	0.047	0.120	0.676	0.211	0.464

Note: Here, I retain all economies in my sample. I use entrepreneurial income as a measure of aggregate profit (EI). *p<0.1; **p<0.05; ***p<0.01.

Table B.3: European Economies: Excluding 2007-2013

	Agg. Profit (% GDP)		Net Foreign Assets (% GDP)		
	(1)	(2)	(3)	(4)	(5)
\overline{tail}	0.415*** (0.092)	7.679*** (1.591)	5.709** (2.141)		
pr				10.595** (3.440)	9.100** (3.446)
FE	Y	Y	Y	Y	Y
Clustering	C	C	C	C	C
Controls			✓		✓
Observations	41	47	47	109	109
R ²	0.510	0.506	0.778	0.383	0.551

Note: Here, I exclude years 2007-2013 from analysis. *p<0.1; **p<0.05; ***p<0.01.

Table B.4: Regression Results: Orbis

	Profit (% Sales)	Net Foreign Assets (% GDP)	
	(1)	(2)	(3)
HHI	0.029+ (0.017)	5.219* (2.594)	
pr			15.137* (7.650)
FE	S, Y	Y	Y
Clustering	CxS	C	C
Observations	2,296	56	56
R ²	0.336	0.247	0.164

Note: Here, I use Orbis to construct aggregate profit shares and Herfindahl-Hirschman Indices of sectoral concentration for a sample of European economies (recommended in [Bajgar et al. \(2020\)](#): Belgium, Germany, Spain, France, Finland, Italy, Portugal and Sweden, 2006-2012). I restrict analysis to industries in the non-financial corporate sector. I follow [Kalemli-Ozcan et al. \(2015\)](#) abstract A.5.3 steps 3 to 10 when cleaning the data. I compute firm-level profit shares as the ratio of *Operating Profit (Loss)* and *Operating Revenue* variables, construct sector-level aggregates using revenues as weights and country aggregates using WIOD sectoral revenue shares. I construct Herfindahl-Hirschman Indices using revenue shares of firms in each country-sector-year group. Due to a small number of observations, I control for year fixed effects in two steps: first, by residualizing the dependent variables with respect to a full set of year dummies; then I use these residuals as dependent variables in regressions. *p<0.1; **p<0.05; ***p<0.01.

B.3 Steady State of the Quantitative Trade Model

There are I economies, K sectors, and N firms in each. Production functions are as before:

$$q_{ikn} = z_{ikn} k_{ikn}^\alpha l_{ikn}^{1-\alpha}.$$

The intermediate varieties are combined into a sectoral good using a CES technology with an elasticity of substitution $\sigma > 1$, and are then combined using Cobb-Douglas technology:

$$Q_i = \prod_K Q_{ik}^{\gamma_{ik}}, \quad \text{where} \quad Q_{ik} = \left[\sum_{j \in \mathcal{I}} \sum_{n \in \mathcal{M}_{ijk}} q_{ijkn}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad \sum_K \gamma_{ik} = 1 \quad \rightarrow \quad P_{ik} Q_{ik} = \gamma_{ik} E_i.$$

Firm prices are now:

$$P_{jikn} = \frac{\sigma}{\sigma-1} \frac{c_{jikn}}{1-s_{jikn}}, \quad (16)$$

where marginal costs of production are market specific:

$$c_{jikn} = \begin{cases} \left(\frac{w_i}{1-\alpha} \right)^{1-\alpha} \left(\frac{r_i + \delta}{\alpha} \right)^\alpha \frac{1}{z_{ikn}} & \text{if sold domestically,} \\ \left(\frac{w_i}{1-\alpha} \right)^{1-\alpha} \left(\frac{r_i + \delta}{\alpha} \right)^\alpha \frac{1}{z_{ikn}} d_{jik} & \text{if sold in } j. \end{cases} \quad (17)$$

Now that capital depreciates, I assume that the firms are required to maintain the capital they borrow by investing to make up for the depreciated stock. The capital is produced by the final good producer. Firm sales shares are now defined for a given market j, k :

$$s_{jikn} = \frac{P_{jikn} q_{jikn}}{P_{jk} Q_{jk}} = \frac{P_{jikn}^{1-\sigma}}{\sum_{i \in \mathcal{I}} \sum_{n \in \mathcal{M}_{jik}} P_{jikn}^{1-\sigma}}. \quad (18)$$

The profit share in each of the markets the firm serves is as before:

$$\pi_{jkn} = \frac{\Pi_{jkn}}{P_{jkn}q_{jkn}} = \frac{P_{jkn}q_{jkn} - c_{jkn}q_{jkn}}{P_{jkn}q_{jkn}} = 1 - \frac{c_{jkn}}{P_{jkn}} = \frac{1}{\sigma} + \frac{\sigma - 1}{\sigma}s_{jkn}.$$

Finally, since there are no fixed costs of operation, $M_{jik} = N$.

Note that the households only consume domestically produced final good, and firms can only buy capital stock locally. Thus, the revenue of the final good producer equals

$$E_i = C_i + \Delta A_i + \delta K_i = w_i L_i + r_i A_i + \Pi_i + \delta K_i,$$

where A_i is the aggregate assets held by the domestic households, ΔA_i is the aggregate household investment, and the right hand side expression plugs in the household income. The GDP, on the other hand, is the sum of the revenue of the varieties goods producers:

$$Y_i = w_i L_i + (r_i + \delta)K_i + \Pi_i.$$

If asset markets are in autarky the two coincide (Case a); else these are distinct (Case b):

$$A_i = K_i + \lambda F_i \quad \rightarrow \quad E_i = Y_i, \quad (19a)$$

$$A_i \neq K_i + \lambda F_i \quad \rightarrow \quad E_i = Y_i \left(1 + r^G \frac{(A_i - K_i)}{Y_i} \right) = Y_i \left(1 + r^G \frac{NFA_i}{Y_i} \right). \quad (19b)$$

The two are linked via the goods market clearing condition of the final good producer:

$$Y_i = \sum_{j \in \mathcal{I}} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{M}_{jik}} s_{jkn} \gamma_{jk} E_j. \quad (20)$$

Firm level factor demands can be aggregated into total factor demand. The labor market

clearing condition is then:

$$w_i L_i = \sum_{j \in \mathcal{I}} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{M}_{jik}} (1 - \alpha) \frac{\sigma - 1}{\sigma} (1 - s_{jiken}) s_{jiken} \gamma_{jk} E_j. \quad (21)$$

The asset market clearing condition differs between autarky and financial liberalization:

$$r_i (K_i + \lambda F_i) = \sum_{j,k,n} \alpha \frac{\sigma - 1}{\sigma} (1 - s_{jiken}) s_{jiken} \gamma_{jk} E_j + r_i \lambda F_i = r_i A_i, \quad (22a)$$

$$r^G \sum_{i \in \mathcal{I}} (K_i + \lambda F_i) = \sum_{j,k,n} \alpha \frac{\sigma - 1}{\sigma} (1 - s_{jiken}) s_{jiken} \gamma_{jk} E_j + r^G \sum_{i \in \mathcal{I}} \lambda F_i = r^G \sum_{i \in \mathcal{I}} A_i, \quad (22b)$$

where F_i is the present discounted value of future profit streams in economy i , $F_i = \frac{\Pi_i}{r_i}$.

Finally, the aggregate profits are as follows:

$$\Pi_i = \sum_{j \in \mathcal{I}} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{M}_{jik}} \Pi_{jiken}^N = \frac{1}{\sigma} + \sum_{j \in \mathcal{I}} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{M}_{jik}} \frac{\sigma - 1}{\sigma} s_{jiken}^2 \gamma_{jk} E_j. \quad (23)$$

Non-financial income of domestic workers and capitalists is age specific:

$$y_s^w = \begin{cases} (1 - \tau^{lab})w & \text{if } t_1 \leq s \leq t_2, \\ T^{soc}w & \text{if } t_2 < s \leq T, \end{cases} \quad y_s^c = \begin{cases} (1 - \tau^{lab})w + \frac{\Pi(1 - \lambda)}{\mu L(T - t_1)} & \text{if } t_1 \leq s \leq t_2, \\ T^{soc}w + \frac{\Pi(1 - \lambda)}{\mu L(T - t_1)} & \text{if } t_2 < s \leq T, \end{cases}$$

where the country subscripts are suppressed for ease of exposition. The budget constraint is standard:

$$c_s^i + a_s^i = y_s^i + (1 + r_i) a_{s-1}^i, \quad \text{where } i \in \{w, c\}. \quad (24)$$

The agent receives the inheritance from their grandparent, so the asset holdings at the start of life are the assets held at the date of death by their grandparent, $a_0^i = a_T^i$. Utility

function for each type is as follows:

$$U = \sum_{s=0}^T \beta^s \frac{(c_s^i/o)^{1-\nu_s}}{1-\nu_s} + k \frac{((a_T^i + \underline{a})/o)^{1-\nu_T}}{1-\nu_T},$$

where $\nu_{s+1} = \nu_{slope} \nu_s$ and all parameters are positive. First order conditions require that

$$c_{s+1}^i = [\beta(1+r_{t+1})]^{\frac{1}{\nu_{s+1}}} o^{\frac{\nu_{slope}-1}{\nu_{slope}}} (c_s^i)^{\frac{1}{\nu_{slope}}}, \quad (25)$$

$$c_T^i = k^{-\frac{1}{\nu_{slope}}} (a_T^i + \underline{a}). \quad (26)$$

Aggregate asset demand in economy i is the sum of assets held by agents of each age and summed across types:

$$A_i = (1-\mu)L_i \sum_{s=0}^{T-1} a_s^w + \mu L_i \sum_{s=0}^{T-1} a_s^c. \quad (27)$$

Definition 1A: (Steady state under financial autarky). The autarkic steady state equilibrium is a set of firm-level shares of each of country-specific sectoral goods markets $\{s\}_{jkn}$, as well as wages $\{w\}_i$, autarkic interest rates $\{r\}_i$, and the levels of GDP $\{Y\}_i$ and expenditure $\{E\}_i$ for each economy such that:

1. Each firm's share of each of the markets it serves satisfies the firm's optimal pricing equations (16) and (17) and the final good producers' demand (18),
2. Each $\{Y\}_i$ and $\{E\}_i$ satisfy the GDP accounting condition (19a) and the final good market clearing condition (20),
3. Each wage in $\{w\}_i$ satisfies the respective labor market clearing condition (21),
4. Each interest rate in $\{r\}_i$ satisfies the asset market clearing condition (22a), where domestic asset demand is determined subject to (23), (24), (25), (26), (27).

Definition 1B: (Steady state under financial integration). The free capital flow steady state equilibrium is a set of firm-level shares of each of country-specific sectoral goods

markets $\{s\}_{jkn}$, the global interest rate r^G , as well as wages $\{w\}_i$, the levels of GDP $\{Y\}_i$ and expenditure $\{E\}_i$ for each economy such that:

1. Each firm's share of each of the markets it serves satisfies the firm's optimal pricing equations (16) and (17) and the final good producers' demand (18),
2. Each $\{Y\}_i$ and $\{E\}_i$ satisfy the GDP accounting condition (19b) and the final good market clearing condition (20),
3. Each wage in $\{w\}_i$ satisfies the respective labor market clearing condition (21),
4. The global interest rate r^G satisfies the global asset market clearing condition (22b), where each country's asset demand is subject to (23), (24), (25), (26), (27).

B.4 Constructing Germany and RoE Economies

WIOD features final- and intermediate expenditures by origin, destination, and sector: X_{ijk}^{FC} and X_{ijnk}^{II} . I use these to construct absorption: $X_{ijk} = X_{ijk}^{FC} + \sum_n X_{ijnk}^{II}$. Next, I discard absorption by all economies but Germany and the seven economies comprising my RoE sample, and add up the latter to form aggregate RoE absorption. Now, $i, j \in \{DE, RoE\}$. I use these to construct GDP Y_i , sectoral expenditure shares γ_{ik} and trade shares Λ_{ijk} as

$$Y_i = \sum_j X_{jik}, \quad \gamma_{ik} = \frac{\sum_{j,k} X_{ijk}}{\sum_{j,k} X_{ijk}}, \quad \Lambda_{ijk} = \frac{X_{ijk}}{\sum_l X_{ilk}}$$

B.5 Calibration of the Quantitative Model

First, I solve for $\sigma, \underline{z}_{ik}, \theta_{ik}, d_{jik}$ that match (i) sector- and country-pair trade shares X_{jik} / X_{jk} , (ii) sector-country real output per worker from WIOD, (iii) the logarithms of sector-level Herfindahl–Hirschman Indices, (iv) average aggregate profit share. I do so as follows:

1. Guess a vector of $\sigma, \underline{z}_{ik}, \theta_{ik}$.

2. Take firm-level productivity draws z_{ikn} from a set of country-sector Pareto distributions parameterized by $\{z_{ik}, \theta_{ik}\}$.
3. Guess a set of firm-level sales, y_{jkn} , and use to solve for implied sales shares s_{jkn} , wages w_i and GDP Y_i .
4. Use firm-level productivities and wages to back out iceberg trade costs that reconcile trade shares in the data and the model.
5. Use trade costs, wages and firm productivities to solve for firm-level sales. Verify if matches the sales guessed in Step 3. If not, update the guess and repeat.
6. When the sales have converged, compute the logarithm of Herfindahl–Hirschman Indices, real output per worker and average aggregate profit shares in the model. Compare with that in the data. If matches, the calibration of the production side is complete; if not, update the guess of $\sigma, z_{ik}, \theta_{ik}$ and repeat from Step 1.

I then calibrate the household side of the model. I do so as follows:

1. Take average Y , π and L from the production side of the model.
2. Guess a vector of $c_1^w, c_1^c, a_T^w, k, \theta, \underline{a}, \nu_{slope}, T^{soc}$.
3. Solve for wage given Y, π, L
4. Use the Euler conditions of both types to obtain consumption levels for each age.
5. Solve for the tax rate that results in balanced government budget given the wage w .
6. Solve for non-financial income for both types.
7. Back out the capitalist bequest consistent with targeted aggregate bequest share.
8. Use budget constraint to solve for asset holdings for both types.

9. Verify that bequests left by both types satisfy the respective bequest optimality conditions and equal assets held at the end of life, global asset markets clear, and that workers and capitalists asset ratio, bequest ratio, and net replacement ratio match the empirical targets. If yes, the calibration of the household side is complete; if not, update the guess of $c_1^w, c_1^c, a_T^w, k, \theta, \underline{a}, v_{slope}, T^{soc}$ and repeat from Step 1.